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Relation between the usual and the entanglement temperature, in a simple quantum system

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HIGHLIGHTS

- We study the entanglement temperature of a two-level atom in an external field.
- We use the Jaynes–Cummings model to study the entanglement temperature.
- In the system, we show that the standard and entanglement temperatures coincide.

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ABSTRACT

We develop a thermodynamical theory to describe the behavior of the entanglement between a single two-level atom with a single mode of the electromagnetic field. The resonant Jaynes–Cummings model is used to study both the entanglement thermodynamics, in particular the entanglement temperature, and its connection with the average number of photons in the optical cavity. We find that this entanglement temperature has a strong dependence with the initial conditions of the atom. We show that the entanglement temperature between the photons and the atom defined in this work is the same temperature obtained within the Jaynes–Cummings model at finite temperature developed in the Thermo-Field Dynamics formalism.

1. Introduction

Concepts such as thermodynamic equilibrium seem impossible to reconcile with the idea of isolated quantum systems since such systems follow unitary evolutions and do not reach a final stationary equilibrium state. Of course, a completely isolated quantum system is an idealization, constructed as a help to understand some phenomena displayed by real systems which may be regarded as approximately isolated. However, we recently [1–4] introduced the concept of temperature for an isolated quantum system which evolves in a composite Hilbert space. To do this we consider the quantum walk on the line (QW) (see Ref. [5] and references therein). The QW is a natural generalization of the classical random walk in the frame of quantum computation and quantum information processing and it is receiving much attention recently [6–9].

In our above mentioned works, we have developed a thermodynamic theory to describe the behavior of the entanglement between the coin and position degrees of freedom of the QW. Henceforth, we call "entanglement temperature" to the temperature associated to the entanglement between different degrees of freedom of an isolated quantum system. We have shown that, in spite of the evolution being unitary, in the QW a steady state is established after a Markovian transient stage. Those studies suggest that, if a quantum dynamics develops in a composite Hilbert space (*i.e.* the tensor product of

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several sub-spaces), then the behavior of an operator that belongs only to one of the sub-spaces may camouflage the unitary character of the global evolution. However it is not clear what is the relation between the usual temperature and the entanglement temperature. This question cannot be answered using the QW because it is an abstract mathematical model; to do this we need a real physical model where both temperatures, the usual and the entanglement temperature, emerge naturally. In order to answer it, we have here chosen one of the simplest and most interesting quantum models, the one known as the Jaynes–Cummings model (JCM) [10,11], that studies the interaction between radiation and matter.

The JCM considers the interaction between a single two-level atom with a single mode of the electromagnetic field. The coupling between the atom and the field is characterized by a Rabi frequency, and a loss of excitation in the atom appears as a gain in excitation of the field. The collapse and the eventual revival of the Rabi oscillation, described by the analytical solution of the JCM, is a direct evidence of the quantum nature of radiation. The use of the JCM has permitted to elucidate basic properties of quantum entanglement as well as some aspects of the relationship between classical and quantum physics. Since it was proposed, the phenomenon has been of permanent interest in the quantum theory of interactions. About 30 years ago it was found that the model exhibits highly non-classic behavior, and the possibility of experimental realizations appeared. The relative simplicity of the JCM and its extensions has drawn much attention in the physics community and, more recently, in the field of the quantum computing [12,13].

Also in the 80s, the Thermo Field Dynamics (TFD) formalism [14,15] was applied to the JCM. The TFD is a method, developed in the 70s by Takahashi and Umezawa [16], for describing Quantum Mechanical systems at finite temperature. Using this method, it is possible to describe the statistical average of an observable at finite temperature as a pure state expectation value. Thus, within the TFD formalism, one does not need to deal with a mixed state, which is a statistical ensemble of pure states at finite temperature. In return for the above advantage, the TFD introduces the so-called tilde particles corresponding to ordinary particles, thus doubling the dimension of the Hilbert space associated to the system. In the TFD method the ordinary particles and the introduced tilde particles represent the dynamical degrees of freedom and the thermal degrees of freedom, respectively.

In the present work we connect, within the JCM, the TFD thermodynamics with the entanglement thermodynamics presented in our previous works [1–3]. The paper is organized as follows. In the next section we review the usual JCM and study the photon thermodynamics in that model. In third section we develop the entanglement thermodynamics for the JCM and study its connection to the TFD-defined temperature. Finally, in the last section we draw some conclusions.

2. Jaynes–Cummings model

We consider the ordinary JCM [11], composed by a single two-state atom in an optical cavity, interacting with a single quantized mode, with frequency ω. The Hilbert space of the JCM has the form of a tensor product

$$\mathcal{H} = \mathcal{H}_{\mathrm{N}} \otimes \mathcal{H}_{\mathrm{A}}, \tag{1}$$

where the photon space, $\mathcal{H}_{\mathrm{N}}$, is spanned by the unitary orthonormal vectors of the photon number state $\{|n\rangle\}$, and the atom space, $\mathcal{H}_{\mathrm{A}}$, is spanned by the two orthonormal quantum states $\{|e\rangle, |f\rangle\}$ that represent the excited and fundamental states of the atom, respectively. Note that the set $\{|n, e\rangle, |n, f\rangle\}$, where $|n, e\rangle = |n\rangle|e\rangle$ and $|n, f\rangle = |n\rangle|f\rangle$, is an orthonormal base in the JCM Hilbert space.

In this model, if the atom excitation frequency ω_a is close to ω, then the system is near the resonance and it is possible to use the rotating wave approximation. In this case, and removing the field vacuum energy, the system Hamiltonian is

$$H = \hbar\omega\, a^\dagger a + \frac{\hbar}{2}\omega_a\sigma_z + \frac{\hbar}{2}g\left(a^\dagger\sigma_- + a\sigma_+\right), \tag{2}$$

where $a^\dagger$ and a are the photon creation and annihilation operators respectively, and act on the photon number state $|n\rangle$. The radiation–matter coupling constant g is fixed by physical considerations such as the cavity volume and the atomic dipole moment. The raising and lowering operators are defined by

$$\begin{aligned}\sigma_+ &\equiv |e\rangle\langle f|,\\ \sigma_- &\equiv |f\rangle\langle e|,\end{aligned} \tag{3}$$

and the z Pauli operator by

$$\sigma_z \equiv |e\rangle\langle e| - |f\rangle\langle f| = [\sigma_+, \sigma_-], \tag{4}$$

and act on the atom states. Then the Hamiltonian, Eq. (2), is such that each photon creation is accompanied by an atomic de-excitation, and each photon annihilation by an atomic excitation. For a given photon number value n, Eq. (2) has the eigenvalues

$$E_\pm(n) = \hbar\omega\left(n + \frac{1}{2}\right) \pm \frac{1}{2}\hbar\Omega_n(\delta), \tag{5}$$

where, for a specific detuning parameter $\delta \equiv \omega - \omega_a$,

$$\Omega_n(\delta) = \sqrt{\delta^2 + (n+1)g^2}, \tag{6}$$