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Magnetic hierarchical deposition

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HIGHLIGHTS

- We introduce a hierarchical deposition model in which blocks interact magnetically.
- The model complements the magnetic Eden model and its variants.
- The model provides an alternative paradigm for tunable surface roughness.
- We demonstrate the robustness of logarithmic fractality under spatial correlations.

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ABSTRACT

We consider random deposition of debris or blocks on a line, with block sizes following a rigorous hierarchy: the linear size equals $1/\lambda^n$ in generation n, in terms of a rescaling factor λ. Without interactions between the blocks, this model is described by a logarithmic fractal, studied previously, which is characterized by a *constant* increment of the length, area or volume upon proliferation. We study to what extent the logarithmic fractality survives, if each block is equipped with an Ising (pseudo-)spin $s = \pm 1$ and the interactions between those spins are switched on (ranging from antiferromagnetic to ferromagnetic). It turns out that the dependence of the surface topology on the interaction sign and strength is not trivial. For instance, deep in the ferromagnetic regime, our numerical experiments and analytical results reveal a sharp crossover from a Euclidean transient, consisting of aggregated domains of aligned spins, to an asymptotic logarithmic fractal growth. In contrast, deep into the antiferromagnetic regime the surface roughness is important and is shown analytically to be controlled by vacancies induced by frustrated spins. Finally, in the weak interaction regime, we demonstrate that the non-interacting model is extremal in the sense that the effect of the introduction of interactions is only quadratic in the magnetic coupling strength. In all regimes, we demonstrate the adequacy of a mean-field approximation whenever vacancies are rare. In sum, the logarithmic fractal character is robust with respect to the introduction of spatial correlations in the hierarchical deposition process.

1. Introduction

Aggregation and deposition phenomena have caught many a scientist's interest since their quantitative physical characterization following the introduction of the famous models of Eden growth and diffusion-limited aggregation (DLA) [1,2]. Our goal in this paper is to merge, along the lines of the magnetic Eden model and variants thereof [3], magnetic

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degrees of freedom and deposition rules. Such models, including the one we propose here, appeal to a multitude of physical circumstances in which order or disorder emerges subject to a local optimization criterion. The paradigm of this behaviour is the selection of a configuration according to the minimization of the energy within a given interaction range around the probe degree of freedom. Many interesting works have appeared, in which the interplay of surface growth, surface roughness and critical phenomena near to or far from equilibrium have been studied, and useful insights have been gathered, complementing our understanding of cooperative phenomena at surfaces and interfaces [4,5].

Within the arena of surface growth models, the concept of fractality is omnipresent. Our second main goal in this paper is to develop a magnetic deposition model on an extraordinary class of fractal surfaces, physically significant but sparsely explored so far. The fractal media we have in mind are the so called logarithmic fractals [6], characterized by an additive, rather than multiplicative, geometrical proliferation rule. While ordinary fractality refers to the proliferation of detail upon magnification in such a manner that the length, area, or volume, increases by a constant factor, logarithmic fractality implies an increment in the form of a constant term. Several works have sketched physical realizations and explored consequences of this unusual, but interesting class of models, for example, in the context of hierarchical deposition [7] or bacterial biofilms [8]. We now turn to the assembly of these concepts with magnetic degrees of freedom in an Ising model spirit and formulate the main question of our research. We ask whether the logarithmic fractal character of the hierarchical deposition landscape is robust with respect to the introduction of magnetic interactions.

2. Model

We start from the hierarchical deposition model introduced in Ref. [6], and assume a one-dimensional surface on which debris is deposited. In the absence of magnetic interactions a block is deposited with probability P. The block size follows a strict hierarchy. The linear size of the block is λ^{-n} in generation n, and λ^n sequential attempts are made to deposit blocks along the unit line $[0, 1]$. Without loss of generality we assume a rescaling factor $\lambda = 3$. In the presence of magnetic interactions each block carries an Ising spin $s = \pm 1$ and the Hamiltonian we employ is of Blume–Emery–Griffiths type [9] (however, without biquadratic interaction)

$$H = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - \mu\sum_i \sigma_i^2. \tag{1}$$

Here, $\langle ij\rangle$ denotes nearest neighbours, J is an exchange energy (>0 for ferro- and <0 for antiferromagnetic interactions) and μ is a chemical potential for deposited blocks. Blocks are nearest neighbours if they have one edge or part of one edge in common (sharing a corner is not enough). Further, the substrate on which the blocks are deposited may or may not be endowed with a spin. For our study we assume a *non-magnetic substrate*. Note that the allowed spin values (± 1 or zero) are independent of the sizes of the blocks. The reduced Hamiltonian reads

$$-\beta H = K\sum_{\langle ij\rangle}\sigma_i\sigma_j + \Phi\sum_i \sigma_i^2, \tag{2}$$

with $\beta = 1/k_BT$ (k_B is the Boltzmann constant and T the absolute temperature), $K = J/k_BT$ and $\Phi = \mu/k_BT$.

Due to the magnetic interactions the deposition probability is not homogeneous but depends on the local field originating from already deposited block spins and on the chemical potential. Consequently, the probability for depositing an "up" block in generation n, is

$$P_i(+) = \exp\left(K\sum_{\langle ij\rangle}s_j + \Phi\right)\Big/ Z_i, \tag{3}$$

and that for depositing a "down" block

$$P_i(-) = \exp\left(-K\sum_{\langle ij\rangle}s_j + \Phi\right)\Big/ Z_i, \tag{4}$$

and that for depositing nothing (a vacancy)

$$P_i(0) = 1/Z_i, \tag{5}$$

with local partition sum

$$Z_i = 2\cosh\left(K\sum_{\langle ij\rangle}s_j\right)\exp(\Phi) + 1, \tag{6}$$

with $i = 1, \ldots, 3^n$.

The homogeneous deposition model is retrieved by setting the magnetic coupling K equal to zero, in which case we obtain the spin-independent probabilities

$$P^{(0)}(+) = P^{(0)}(-) = \exp(\Phi)/Z^{(0)} \equiv P/2, \tag{7}$$