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A graph-based approach to inequality assessment☆

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HIGHLIGHTS

- We study inequality measurement on graph theory framework.
- Graph represents the set of connections between income sources for the population.
- Different policies can be implemented to reduce inequality based on the graph structure.
- A multifactorial inequality index is introduced to outline an example.

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ABSTRACT

In a population consisting of heterogeneous types, whose income factors are indicated by nonnegative vectors, policies aggregating different factors can be represented by coalitions in a cooperative game, whose characteristic function is a multi-factor inequality index. When it is not possible to form all coalitions, the feasible ones can be indicated by a graph. We redefine Shapley and Banzhaf values on graph games to deduce some properties involving the degrees of the graph vertices and marginal contributions to overall inequality. An example is finally provided based on a modified multi-factor Atkinson index.

1. Introduction

Issues of economic inequality have pushed their way back into the national and global conversation as one of the crucial point to guarantee sustainability of the system.

An issue that has received scanty attention in the measurement of inequality is how income sources connect to each other. Consequently, the role that such connections play in influencing the level of inequality in a distribution still remains unexplored. The real world examples are numerous.

Connections among sources are, for instance, important in the sphere of policy for employment. Taxpayers who receive income from more than one source, e.g. employment or pension, should keep in mind that the employees' tax deducted by the respective employers or pension funds might not be enough to cover their final tax liability on assessment. Deciding whether separating or not such sources does make a difference in raising the eligible retirement age or simply reducing social security benefits.

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Still, when a married couple decides to go through separation and the separation is likely to be permanent, different country-based taxation options are available according to the existing links in the maintenance payments. If a couple is considered as two single persons, there will be no change in their tax assessment if they subsequently break up, whereas if they are taxed under separate assessment, their income up to the date of separation is assessed in the usual way and they can transfer between them any unused tax credits and rate bands that apply.[1]

Imagine now a connection among welfare policies. By making the receipt of unemployment benefits conditional on engagement in an incentive-enhancing activity (e.g., work under state-sponsored employment schemes or participation in work-site-based training programs), a government may engineer a revision of the policies in order to induce agents to seek work raising public good provision.

From an international perspective, the intersection of commercial income sources across countries necessarily caused the intersection of other forms of regulating incomes. International trade, for example, forces a confrontation and rationalisation of international tax rules, in the interests of international trade but also the interests served to fund government expenditures on welfare states or to direct forms of economic activity considered beneficial for a healthy domestic economic and social environment.

Given the importance of income sources in reality, investigating their linkages as well as mechanisms geared at reducing inequality is an alternative way of interpreting a process, where sources shape the infrastructure of the income distribution. A model incorporating the above features first requires the form of an income network. Second, such perspective complements the view of the traditional graph structure as the transmission scheme of information. For instance, such information may concern discrimination between labor and financial income, personal income taxation and, more in general, a graph's links can be viewed as the suppliers of heterogeneous types of support for policy implementations. Differently from standard cooperative games, a graph portrays an environment where some combinations of sources are not possible, meaning that some links among the sources are excluded, hence not all sources can be associated to all the remaining ones.

In this paper, we take a step towards this analysis. We present a graph (or a network) setting, where vertices represent income sources and an edge connecting two vertices represents the feasibility of a coalition between them.[2] The complexity of graphs is well-known, therefore we adopt a soft approach relying on some standard characteristics of these structures. Since our graphs are not complete, we are going to establish a distinction among different kinds of coalitions: non-feasible, feasible and complete, feasible and incomplete. This allows us to introduce a new decomposition of the Shapley and Banzhaf indices taking into account the above coalitions in differentiated ways.

After briefly outlining some useful tools and redefining the Shapley and Banzhaf indices on graphs, we construct a class of graphs to derive specific properties of the marginal contributions to inequality. Within this class, a clear distinction is made between the sources (and consequently their related policies), which can be all freely connected, and the ones combined with a limited group of other policies, due to political or budgetary constraints.

The paper is organized as follows. In Section 2, a literature review about the recent frontier of inequality measurement is presented and some differences existing between past methodologies and our one are pinpointed. Section 3 describes the model with some definitions of graph theory. Section 4 describes the cooperative game setup and the main theoretical findings on Shapley and Banzhaf allocation rules, whereas Section 5 identifies a class of graphs to which the previous findings can be applied. Section 6 contains an example based on a multi-factorial inequality index in Atkinson fashion. Concluding remarks are collected in Section 7.

2. Literature review

The traditional procedures for the decomposition of income inequality are due to Atkinson [2], Blackorby et al. [3], Shorrocks [4,5] and Lerman and Yitzhaki [6] in the '80s.

Among others, Shorrocks [4] examines decomposition of inequality by income sources such as earnings, investment incomes and transfer payments. He shows that a broad class of inequality measures can be decomposed into components reflecting the sum of all sources' contributions. This is called a *natural decomposition* of inequality and gives the opportunity to derive an infinite number of decompositions without restrictions. This framework is also investigated by Lerman and Yitzhaki [6], who exploit this property proposing a covariance formula of the Gini coefficient á la Fei et al. [7]. Similar analysis for welfare foundation had already been developed by Atkinson [2] in 1970.

Any standard analysis to gauge income inequality simply provides a certain measurement of the level of inequality in the society. By looking at different dimensions (e.g. comparisons among wealth classes, intensity of inequality, redistribution policies), we can extend our view into a more comprehensive treatment, thereby adding immeasurable value to the related literature.

In particular, some economists and econophysicists have been in the quest of deriving alternative procedures by molding mathematical and statistical properties of the existing tools into helpful forms for distributional and sensitive analysis.

[1] For the remainder of the tax year after separation, each spouse counts as a single person, while individual tax credit applies to his or her income. Instead, under joint assessment, one spouse is accountable for tax purposes of the other assessable spouse.

[2] A recent contribution about inequality measurement and a network structure has been published by Kets et al. in 2011 [1].