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Hyperbolic mapping of complex networks based on community information

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HIGHLIGHTS

- A community intimacy index is proposed to measure the community relationship.
- A community ordering algorithm is proposed based on the community intimacy index.
- Improved hyperbolic mapping method *CHM* is proposed.
- The new method improves accuracy and running time complexity compared to the state-of-the-art.

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ABSTRACT

To improve the hyperbolic mapping methods both in terms of accuracy and running time, a novel mapping method called Community and Hyperbolic Mapping (*CHM*) is proposed based on community information in this paper. Firstly, an index called Community Intimacy (*CI*) is presented to measure the adjacency relationship between the communities, based on which a community ordering algorithm is introduced. According to the proposed Community-Sector hypothesis, which supposes that most nodes of one community gather in a same sector in hyperbolic space, *CHM* maps the ordered communities into hyperbolic space, and then the angular coordinates of nodes are randomly initialized within the sector that they belong to. Therefore, all the network nodes are so far mapped to hyperbolic space, and then the initialized angular coordinates can be optimized by employing the information of all nodes, which can greatly improve the algorithm precision. By applying the proposed dual-layer angle sampling method in the optimization procedure, *CHM* reduces the time complexity to $O(n^2)$. The experiments show that our algorithm outperforms the state-of-the-art methods.

1. Introduction

Due to the limited information that network topology alone contains, the traditional methods that only concerning network topology are difficult to thoroughly study the issues about the functions, dynamical processes and evolution of the complex networks [1]. The hyperbolic geometry of complex network, which is becoming an emerging research field, has

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shown many advantages in network structure, functions, and evolution [2–6]. Therefore, how to map the network to its corresponding hyperbolic space is becoming a critical issue.

The network model built on the hyperbolic space can generate networks [5–7] whose characteristics are surprisingly similar to the actual networks. The high-level explanation of this phenomenon is that the complex networks exhibit hierarchical, tree-like organization [8] and hyperbolic geometry is exactly a kind of tree geometry. The recent studies have proposed several different hyperbolic space models, including *Newtonian-*S^1 [5], *Einsteinian-*H^2 [6] and Popularity $\times$ Similarity Optimization (*PSO*) [7], and the corresponding mapping methods: *MLE wrapper* [9] and *HyperMap* [10].

MLE wrapper algorithm maps Internet on the Autonomous System (*AS*) level [11] to a two-dimensional hyperbolic space by taking advantage of the isomorphism between *Newtonian-*S^1 model and *Einsteinian-*H^2 model to calculate the expectation degree and angular coordinates of the nodes. They found that the mapped network in the hyperbolic space was highly navigable, which meant that there was a very high success rate when transmitting the routing packets with greedy strategy, and it was able to detect the soft communities, which meant that the *AS*s belonging to a same country except USA were mapped into a same angular sector in the 2D hyperbolic space. While *HyperMap* algorithm, based on *PSO* model, maps the network into hyperbolic space by replaying the growth process of the network in the hyperbolic space. In addition to the high navigability and soft communities, the mapped network also shows good ability to predict the missing links of the network. It is worth mentioning that, *PSO* model considers node popularity and similarity as driving force of network growth, and portrays the popularity with node radial coordinate r and the similarity with node angular coordinate θ. *PSO* model can simulate almost all types of network evolution models by adjusting evolutionary mechanism. Thus, theoretically, the suitable mapping methods based on *PSO* model can map most types of networks.

Even though the existing mapping methods can accurately map the networks into the hyperbolic space, they are still faced with some common problems. Firstly, the time complexity of the existing mapping models are $O(n^3)$ [9,10], where n is the total number of nodes in the network. It means that the mapping methods are difficult to be applied in practice, especially in the large networks with hundreds of thousands of nodes. Secondly, the mapping precision of the existing methods depends on the nodes with high degree value, but it is difficult to precisely estimate all high degree value nodes' angular coordinates θ [10]. Thus, Papadopoulos et al. [10] discussed some correction steps, by trying to re-compute improved angles for the first nodes, considering not only the connections to their previous nodes, but also connections to nodes that appear later. Therefore, how to improve the mapping efficiency and accuracy is a key issue that must be addressed when designing hyperbolic mapping method.

The above accuracy problems are caused by mapping process used by existing mapping methods, which map the nodes to hyperbolic space one by one. They use the information of mapped nodes to compute the angular coordinate of a new node to be mapped, while many nodes with effective information are not mapped yet. Therefore, because of the imperfect information, the computed angular coordinate of new node is less accurate which cannot be offset in the subsequent optimization procedure completely. To solve the above accuracy problem, this paper proposes a mapping method: Community and Hyperbolic Mapping (*CHM*). Based on the *PSO* model and its coordinate system, *CHM* firstly takes advantage of community structures of complex network to preliminarily estimate the similarity between nodes. Secondly, *CHM* orders the communities by measuring the proximity relation of communities and then maps the ordered communities into the corresponding sectors in hyperbolic space, which means that the sectors are also ordered based on the order of the mapped communities and the nodes of each community are mapped into a same sector. Thirdly, *CHM* randomly initializes the angular coordinates of the nodes in the same sector that the nodes belong to, and then uses the maximum likelihood estimation method to precisely optimize the angular coordinates of the nodes. The application of community initialization methods guarantees that the initial angles of most nodes are close to their real values, which helps to solve the mapping accuracy problem because all nodes have been mapped and their information can be employed in the optimization procedure. What is more, we proposed a numerical optimization method called dual-layer angle sampling to optimize the angular coordinates in the optimization procedure of *CHM*, which reduced the computational complexity to $O(n^2)$. Compared to *HyperMap*, *CHM* yields a good balance between the time complexity and mapping accuracy.

The rest of the paper is organized as follows. In Section 2, we propose the Community-Sector hypothesis. Section 3 presents the mapping method *CHM*. Section 4 experimentally validates the mapping accuracy of *CHM* on synthetic and real networks. We discuss and analyze the results in Section 5. Then the conclusions are given in Section 6.

2. Community-sector hypothesis

The hyperbolic space described in this paper refers to a two-dimensional hyperbolic disc.

Community-Sector hypothesis: In this paper we assume that network communities can be well separated into distinct sectors in the underlying hyperbolic disc, which means that nodes belonging in a community can be mapped into the community's sector in the disc. We ignore possible overlaps between communities, and we call this assumption the Community-Sector hypothesis.

We analyze the rationality and correctness of the above Community-Sector hypothesis below.

A community of complex network is usually composed of nodes that are more similar in structure and property. Community structure depicts the local clustering characteristics and heterogeneous distribution of edges of the network [12]. And in hyperbolic space, the difference between angular coordinates of nodes characterize the similarity