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Influence of competition in minimal systems with discontinuous absorbing phase transitions

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ABSTRACT

Contact processes (CP's) with particle creation requiring a minimal neighborhood (restrictive or threshold CP's) present a novel sort of discontinuous absorbing transitions, that revealed itself robust under the inclusion of different ingredients, such as distinct lattice topologies, particle annihilations and diffusion. Here, we tackle on the influence of competition between restrictive and standard dynamics (that describes the usual CP and a continuous DP transition is presented). Systems have been studied via mean-field theory (MFT) and numerical simulations. Results show partial contrast between MFT and numerical results. While the former predicts that considerable competition rates are required to shift the phase transition, the latter reveals the change occurs for rather limited (small) fractions. Thus, unlike previous ingredients (such as diffusion and others), limited competitive rates suppress the phase coexistence.

1. Introduction

The usual contact process [1] is probably the simplest example of system presenting an absorbing phase transition. It is composed of two subprocesses: "spontaneous" annihilation and "catalytic" particle creation, in which new species are created only in empty sites on the neighborhood of at least one particle. Despite the lack of an exact solution, its phase transition and critical behavior are very well known and belong to the robust directed percolation (DP) universality class [2–4]. Many generalizations of its rules can be extended not only for theoretical purposes, but also for the description of a large variety of systems in the framework of physics [5,6], chemistry [7,8], ecology [9] and others. In these cases, the competition among dynamics leads to several new findings. In some cases [5,10], the competition between particle hoping (diffusion) and annihilation of three adjacent particles (instead of a single particle) is responsible for a reentrant phase diagram and a stable active phase for extremely low activation rates. In other examples [11], by allowing particles to have different creation rates with respect to their first and second neighbors, the competition is responsible for the appearance of an active asymmetric phase with spontaneous breaking symmetry. Also, when particles interact in a symbiotic manner [12], two active symmetric phases, in which only one species is present, emerge. An interesting generalization is the called restrictive (threshold) CPs, in which the phase transition changes from continuous to discontinuous for $d \geq 2$. They are similar to the usual CP, but one requires at least two particles for creating a new species (in the original CP at least one particle is needed). Different restrictive models have revealed that the phase transition remains first-order by including

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other sorts of creation [8,13–16], annihilation rules [16] and also different lattice topologies [17]. Besides, a very recent study [18] has claimed that the particle diffusion does not suppress the phase coexistence, in partial contrast with results obtained from stochastic differential equation approaches [9].

In order to enhance the understanding about the robustness of discontinuous absorbing phase transitions presented in such simple (minimal) models, here we extend the study undertaken in Ref. [18] by addressing the competition between the above restrictive and standard dynamics (that describes the usual CP). In the last case, the transition belongs to the usual and robust directed percolation (DP) universality class. More specifically, the particle creation requiring at least one and two nearest neighbor particles is chosen with distinct (but complementary) probabilities. Two distinct restrictive rules will be considered. Although the phase transition is expected to be continuous (discontinuous) in the extreme regimes of less (more) frequent restrictive interactions [14,15], our study focuses on answering the following questions: (i) is the phase coexistence suppressed for the inclusion of limited fraction of non-restrictive interactions? (ii) or on the contrary, only large rates are required for shifting the phase transition? (iii) Finally, how does the difference of models influence the phase transition and corresponding tricritical points (separating phase coexistence from critical transition)?

Models will be investigated in the framework of mean field theory (MFT) and numerical approaches, which lead (as will be shown further) to conclusions in partial agreement. While the MFT predicts the inclusion of considerable fraction of non-restrictive interactions is needed for shifting the phase transition, numerical simulations show the suppression occurs for limited small rates. A second contribution concerns the establishment of precise approaches for characterizing the discontinuous transitions. Although the critical exponents for DP phase transitions are well known, no established scaling behavior is known for the discontinuous case. As it will be shown, our methodology clearly distinguishes continuous from discontinuous transitions, reinforcing previous claims about the existence of a common finite-size scaling for the latter case [16,18,19].

This paper is organized as follows: In Section 2 we define the models and mean-field predictions are presented in Section 3. Section 4 shows the numerical results and conclusions are drawn in Section 5.

2. Models

Systems are defined on a square lattice of size L and each site has an occupation variable σ_i that assumes the value 0 or 1 depending on whether it is empty or occupied, respectively. The dynamics is composed of the following ingredients: particle annihilation and creation requiring a minimal neighborhood nn of at least 1 and 2 particles. More precisely, particles are annihilated with rate α and with probabilities p and $1-p$ the particle creation requires at least $nn \geq 1$ and $nn \geq 2$ adjacent particles, respectively. Here, we consider two different rules for the second creation subprocess. In the first case (rule A), the creation occurs with rate $nn/4$ [14], whereas in the latter (rule B) the creation is always 1 (provided $nn \geq 2$ in both cases) [15]. Thus, while for the rule A particles are created with rate proportional to the number of their nearest neighbors, it is independent of nn for the rule B.

The extreme cases $p = 0$ and $p = 1$ reduce themselves to the restrictive models investigated in Refs. [14,15] (rules A and B) and the usual CP, respectively. The transitions are discontinuous and continuous yielding $\alpha_0 = 0.2007(6)$ (rule A and $p = 0$) [14], $\alpha_0 = 0.352(1)$ (rule B and $p = 0$) [15] and $\alpha_c = 0.60653(1)$ [20] for $p = 1$. In all cases, the order parameter is the particle density ρ, in such a way that $\rho = 0$ in the absorbing state and $\rho \neq 0$ in the active phases.

3. Mean-field analysis

Since all above models present no exact solution, the first inspection over the effect of competition can be undertaken by performing mean-field analysis (MFT). Starting from the master equation, we derive relations for appropriate quantities and truncate the associated probabilities. In the first level of approximation (one-site mean-field), the generic probability $P(\sigma_0, \sigma_1, \ldots, \sigma_{n-1})$ is rewritten as a product of one-site probabilities, in such a way we have $P(\sigma_0, \sigma_1, \ldots, \sigma_{n-1}) = P(\sigma_0)P(\sigma_1)\ldots P(\sigma_{n-1})$. Due to the relation $\sum_{\sigma_i=0}^{1} P(\sigma_i) = 1$ only one relation is sufficient for the analysis.

In order to obtain improved results, we include correlation of two sites. This can be done by performing the pair mean-field approximation that consists of rewriting the n-site probabilities ($n > 2$) as products of two-site probabilities yielding

$$P(\sigma_0, \sigma_1, \ldots, \sigma_{n-1}) \simeq \frac{P(\sigma_0, \sigma_1)P(\sigma_0, \sigma_2)\ldots P(\sigma_0, \sigma_{n-1})}{P(\sigma_0)^{n-2}}. \tag{1}$$

In this case, two equations are required to calculate the system properties. By identifying the system density ρ as the one-site probability $\rho = P(1)$ and considering the two-site correlation $u = P(01)$, from the previous models' rules we obtain the following relations

$$\frac{d\rho}{dt} = (1-p)[2P(01100) + P(01010) + 3P(01011) + P(01111)] + pP(01) - \alpha P(1), \tag{2}$$

$$\frac{du}{dt} = (1-p)\left[-\frac{3}{2}P(01011) - P(01111)\right] + \frac{p}{2}[-3P(011) + P(01)] - \alpha\, P(01) + \alpha\, P(11), \tag{3}$$