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Dynamic scaling behaviors of linear fractal Langevin-type equation driven by nonconserved and conserved noise

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HIGHLIGHTS

- A generalized linear fractal Langevin-type equation driven by nonconserved and conserved noise is proposed.
- The scaling behaviors of this equation are investigated theoretically by scaling analysis.
- Corresponding dynamic scaling exponents are very consistent with the numerical results of simulation.

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ABSTRACT

In order to study the effects of the microscopic details of fractal substrates on the scaling behavior of the growth model, a generalized linear fractal Langevin-type equation, $\partial h/\partial t = (-1)^{m+1}\nu\nabla^{mz_{rw}}h$ (z_{rw} is the dynamic exponent of random walk on substrates), driven by nonconserved and conserved noise is proposed and investigated theoretically employing scaling analysis. Corresponding dynamic scaling exponents are obtained.

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The kinetic roughening of surfaces and interfaces under non-equilibrium conditions has been a subject of great interest in the last two decades, due to their relations to various physical phenomena such as crystal growth, bacterial growth, molecular beam epitaxy (MBE), fluid in porous, and fracture cracks among others [1–4]. A common feature of many interfaces observed experimentally or in discrete growth models is that their roughening follows simple scaling laws [5]. The morphology and dynamics of a rough interface can be characterized by the surface width, $W(L, t)$, that scales as

$$W(L,t) \equiv \langle \overline{[h(\mathbf{x},t) - \overline{h}(t)]^2} \rangle^{1/2} \tag{1}$$

where L is the linear size and d is the dimension of the substrate. $h(\mathbf{x}, t)$ is the local height variable of the interface, and $\overline{h}(t)$ is the mean height of the interface at time t. In many cases, starting from an initially flat substrate, the surface width is observed to satisfy the dynamic scaling form of Family–Vicsek [5]

$$W(L,t) = t^{\beta} f(L/t^{1/z}), \tag{2}$$

with the scaling function $f(u)$ behaves as $f(u) \sim u^{\alpha}$ if $u \ll 1$, and $f(u) \sim const$ if $u \gg 1$. The roughness exponent α and the dynamic exponent z determine the asymptotic behavior of growing interfaces on a large distance and long time scale. The ratio $\beta = \alpha/z$ is called growth exponent and describes the short time behavior of the surface.

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One of the widely used methods of getting these scaling exponents is applying a numerical or analytical approach to the associated stochastic evolution equations (usually Langevin-type) that describe the interface growth processes. Diffusion, with additive noise, is the fundamental element. Generally, the $d+1$-dimensional Langevin-type equation can be described as

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\Psi(\nabla h)+\eta(\mathbf{x},t), \tag{3}$$

where the function $\Psi(\nabla h)$ defines a particular model and incorporates the relevant symmetries and conservation laws. Surface growth is driven by an external noise $\eta(\mathbf{x},t)$, which represents the influx of particles in deposition processes. The noise, usually, has zero mean and correlations

$$\left\langle \eta\left(\mathbf{x},t\right)\eta\left(\mathbf{x}',t'\right)\right\rangle = 2D\delta\left(\mathbf{x}-\mathbf{x}'\right)\delta\left(t-t'\right), \tag{4}$$

for nonconserved type, or

$$\left\langle \eta_c\left(\mathbf{x},t\right)\eta_c\left(\mathbf{x}',t'\right)\right\rangle = -2D_c\nabla^2\delta\left(\mathbf{x}-\mathbf{x}'\right)\delta\left(t-t'\right), \tag{5}$$

for conserved one, with D and D_c specify the noise amplitude for nonconserved noise and conserved noise, respectively. A seminal example of this kind of stochastic dynamic equations is the well-known Edwards–Wilkinson (EW) equation [6], which is given as

$$\frac{\partial h}{\partial t}=\nu\nabla^2 h+\eta\left(\mathbf{x},t\right). \tag{6}$$

The first term on the right hand side describes the relaxation of interface caused by a surface tension ν. Other equations, such as the Kardar–Parisi–Zhang (KPZ) equation [7], were also proposed. The analytical tools now widely used to investigate the scaling behaviors of these Langevin-type equations are the scaling analysis [8], the dynamic renormalization group technique [9,10], the mode coupling method [11], and so on.

Among previous theoretical investigations of continuum equations, as well as numerical simulations of discrete atomistic models, much more were performed on regular or Euclidean substrates with integer dimension, however, less were devoted to fractal substrates. As a result, there is no simple and clear understanding about the interplay between the dynamical growth rules of the system and the self-similarity of fractal structures until the recent works [12–20].

It was found that the dynamic scalings on fractal substrates were not consistent with the known results of the EW equation. Lee and Kim [12] argued that the term $\nabla^2 h$ in the EW equation has symmetries under inversion and rotation in the $\mathbf{x}$ space. However, on a fractal substrate, the diffusion is anomalous and the dynamic exponent of random walks z_{rw}, defined by the root mean-square end-to-end distance via $\langle R^2\rangle \sim t^{2/z_{rw}}$, is larger than 2. Also, no symmetry exists under either inversion or rotation on usual fractal substrates. Therefore, the term associated with diffusion $\nabla^{z_{rw}}h$ was assumed, and the fractal Langevin equation

$$\frac{\partial h}{\partial t}=\nu\nabla^{z_{rw}} h+\eta\left(\mathbf{x},t\right) \tag{7}$$

was introduced [12]. Utilizing the scaling analysis, this fractal equation can be solved exactly, leads to $\beta = 1/2 - d_s/4$, $\alpha = d_f(1/d_s - 1/2)$, $z = 2d_f/d_s$, and the scaling relation $2\alpha + d_f = z$. d_s is the spectral dimension defined by the density of normal modes on fractal lattices via $\rho(\omega) \sim \omega^{d_s-1}$ and d_f is the dimension of the fractal substrate. Here the dynamic exponent of random walks on a fractal substrate is given as $z_{rw} = 2d_f/d_s$. The equilibrium restricted solid-on-solid (RSOS) model and the Family model on fractal substrates were studied [13,14], and the results proved the universality of this fractal equation.

In one of their recent works, Kim and co-workers [15] performed numerical simulations on the surface structures of the equilibrium restricted curvature (ERC) model on a Sierpinski gasket substrate and obtained the results that $\alpha \approx 1.54$, $\beta \approx 0.323$ and $z = \alpha/\beta \approx 4.78$. The scaling exponents derived satisfied the relations $2\alpha + d_f \approx z$ and $z \approx 2z_{rw}$ very well [15]. To describe this discrete model on fractal substrates, they introduced the fractional Langevin equation [15], which is called the fractal Mullins–Herring (MH) equation, written as

$$\frac{\partial h}{\partial t}=-\nu\nabla^{2z_{rw}} h+\eta(\mathbf{x},t). \tag{8}$$

Employing the scaling analysis as used previously, the linear fractal MH equation can be solved exactly by giving $\beta = 1/2 - d_f/4z_{rw}$, $\alpha = (2z_{rw} - d_f)/2$, and $z = 2z_{rw}$. It can be seen that there exists a scaling relation $2\alpha + d_f = z$. After that, the validity of the fractional MH equation and the scaling relation were confirmed by numerical work [16].

In this paper, to probe the effects of the microscopic details of fractal substrates on the scaling behavior of the growth model deeply, the generalized linear fractal Langevin-type equations, driven by both nonconserved and conserved noise, are proposed and investigated theoretically based on scaling analysis. The corresponding dynamic scaling exponents are obtained.

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