Physica A 451 (2016) 490–501

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Pricing turbo warrants under mixed-exponential jump diffusion model

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HIGHLIGHTS

- Analytical pricing formula of turbo warrant under mixed-exponential jump diffusion model is derived.
- Numerical Laplace inversion examples are provided based on the extended Euler method.
- The performance of inversion results are evaluated via comparing with the prices from Monte Carlo simulation.
- The numerical simulation confirms that jump risk should not be ignored in the valuation of turbo warrants.

ARTICLE INFO

Article history:
Received 11 September 2015
Received in revised form 10 November 2015
Available online 9 February 2016

Keywords:
Exotic option
Turbo warrants
Mixed-exponential jump diffusion
Laplace transform
Euler method

ABSTRACT

Turbo warrant is a special type of barrier options in which the rebate is calculated as another exotic option. In this paper, using Laplace transforms we obtain the valuation of turbo warrant under the mixed-exponential jump diffusion model, which is able to approximate *any* jump size distribution. The numerical Laplace inversion examples verify that the analytical solutions are accurate. The results of simulation confirm the argument that jump risk should not be ignored in the valuation of turbo warrants.

1. Introduction

The word, *turbo warrant*, firstly appeared in Germany at the end of 2001 as the name of a warrant having a knock-out level at the strike price. This kind of instrument is essentially a knock-out barrier option. A more interesting situation appears when the turbo warrant is strictly in the money and a rebate is paid if the barrier is hit. Such contracts were introduced at the end of February 2005 by Société Generale [1,2]. Société Generale listed the first 40 turbo warrants on the Nordic Growth Market (NGM) and Nordic Derivatives Exchange. In June 2006, the Hong Kong Exchange and Clearing Limited (HKEx) introduced callable bull/bear contracts (CBBC), which were essentially turbo warrants.[1]

Turbo warrants are attractive because it is believed that they have a low *vega*, meaning that their prices are much less sensitive to the implied volatility. This view is based on the classical Black–Scholes diffusion model in which the volatility is constant [3]. However, when stochastic volatility is taken into account, turbo warrants could be very sensitive to the shape of the volatility smile and this sensitivity is model-dependent [4].

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[1] For more information, see the website of HKEx: http://www.hkex.com.hk/eng/index.htm.

http://dx.doi.org/10.1016/j.physa.2015.12.158

As well as the stochastic volatility models, jump diffusion models are among the most popular approaches to model the volatility smile especially for short-term contracts. Wong and Lau [2] evaluate turbo warrants under the double exponential jump diffusion model (DEM) [5–7]. Besides, more general models have been proposed to better capture the features of underlying financial data, such as phase-type jump diffusion models (PHM) [8], and truncated levy models [9–12]. In this paper, we consider the pricing problem of turbo warrants under the mixed-exponential jump diffusion model (MEM) [13].

MEM is under our consideration for three reasons. First, MEM extends existing models, such as DEM and hyper-exponential exponential jump diffusion models (HEM) [14–16]. Second, the mixed-exponential distribution is able to approximate *any* distribution of jumps arbitrarily closely, including any discrete distribution, the normal distribution, and various heavy-tailed distributions such as Gamma, Weibull, and Pareto distributions.[2] Third, MEM allows analytical tractability for path-dependent options and leads to analytical solutions for Laplace transforms [13]. In pricing exotic options, such as turbo warrants, lots of difficulties are related with the first passage time and the *overshoot*, the amount that asset prices cross over a certain level when jumps are allowed. These issues can be well handled under the MEM.

The analytical valuation of turbo warrant under MEM is obtained via the Laplace transform approach. An extended Euler algorithm is applied to numerically invert those solutions [18]. For comparison, we use the results of Monte Carlo simulation as benchmarks to verify the accuracy of the analytical solutions. Sensitivity analysis shows that turbo warrants are less sensitive to the jump intensity than the corresponding vanilla call options, but are much more sensitive to the jump asymmetry. The comparison of valuations under Geometric Brownian motion environment and MEM setting further suggests that jump risk should not be ignored in the valuation of turbo warrants. In some circumstances, the pricing error is larger than 15%.

The remainder of the paper is organized as follows. Section 2 gives the risk-neutral pricing formula of turbo warrant which is a model-free representation. Section 3 introduces the MEM model and derives the analytical solutions of turbo warrants via Laplace transform. Section 4 provides numerical examples. Section 5 concludes the paper.

2. Turbo warrants

Turbo warrant is a special type of knock-out barrier options in which the rebate is specified as another exotic option. Denote the underlying asset price as S_t and the barrier price as B. A turbo call warrant with strike price K pays the holder $(S_T - K)^+$ at maturity T if the barrier price $B \geq K$ has not been passed at any time prior to the maturity. Denote the first time the underlying asset price S_t hits the barrier as τ_b, i.e., $\tau_b = \inf\{t | S_t \leq B = S_0 e^b\}$. If $\tau_b \leq T$, the option is knocked out and a new option enters into force. This new option is usually a call option on $m_{\tau_b}^h = \min_{\tau_b \leq t \leq \tau_b + h} S_t$, with the same strike price, K, and a short time to maturity, h.

Precisely, the price of turbo call option can be represented as

$$TC(t, S_t) = \mathbb{E}[e^{-rT}(S_T - K)^+ I_{\{\tau_b > T\}} + e^{-r(\tau_b + h)}(m_{\tau_b}^h - K)^+ I_{\{\tau_b \leq T\}} | \mathscr{F}_t], \tag{2.1}$$

where r is the risk-free interest rate, and the conditional expectation represents the risk-neutral expectation conditional on the information up to time t.

It can be seen from (2.1) that a turbo call warrant consists of two parts. The first part is a down-and-out call (DOC) option with no rebate and the second part is a down-and-in lookback (DIL) option. Thus, we obtain the following proposition.

Proposition 2.1. *When $t < \tau_b$, the risk-neutral pricing formula of the turbo call warrant is given by*

$$TC(t, S_t) = DOC(t, S_t) + DIL(t, S_t, h) \tag{2.2}$$

where

$$DOC(t, S_t) = \mathbb{E}[e^{-rT}(S_T - K)^+ I_{\{\tau_b > T\}} | \mathscr{F}_t], \tag{2.3}$$

and

$$DIL(t, S_t, h) = \mathbb{E}[e^{-r(\tau_b + h)}(m_{\tau_b}^h - K)^+ I_{\{\tau_b \leq T\}} | \mathscr{F}_t]. \tag{2.4}$$

3. Pricing turbo warrants under MEM

MEM [13] is a jump diffusion model whose jump size has a mixed-exponential distribution—a weighted average of several exponential distributions. This feature ensures the model flexibility. Since a turbo warrant can be decomposed into a DOC option and a DIL option, see Proposition 2.1, we obtain their pricing formulas separately and then put the results together to get the analytical solutions for turbo warrants.

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2 An important advantage of MEM over HEM is that the latter cannot approximate the distributions that are not completely monotone [17,13].