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Long memory and multifractality: A joint test

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HIGHLIGHTS

- Hypothesis tests for the multifractal model of asset returns are developed.
- Conventional tests reject the null hypothesis of no long memory too frequently.
- Long memory and multifractality are estimated jointly using exchange rate data.
- Most of the exchange rate returns series are most appropriately characterized by a variant of the MMAR that applies a multifractal time-deformation process to NIID returns.
- None of the exchange rate returns series exhibits evidence of long memory.

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ABSTRACT

The properties of statistical tests for hypotheses concerning the parameters of the multifractal model of asset returns (MMAR) are investigated, using Monte Carlo techniques. We show that, in the presence of multifractality, conventional tests of long memory tend to over-reject the null hypothesis of no long memory. Our test addresses this issue by jointly estimating long memory and multifractality. The estimation and test procedures are applied to exchange rate data for 12 currencies. Among the nested model specifications that are investigated, in 11 out of 12 cases, daily returns are most appropriately characterized by a variant of the MMAR that applies a multifractal time-deformation process to NIID returns. There is no evidence of long memory.

1. Introduction

The statistical properties of financial asset returns are of key importance for finance theory, and for portfolio and risk management. The econophysics literature has grown rapidly in volume over the past two decades, and fractal models have been employed to explain empirical regularities at odds with mainstream financial economics theory, such as power-laws and self-similarity [1], long memory in returns and volatility [2–4], and multifractality [5]. Multifractality in financial returns data has been interpreted as evidence of financial market inefficiency [6], or herding behaviour in financial markets [7].

The multifractal model of asset returns (MMAR) is capable of accommodating exceptional events (large shocks), and can represent either normal or non-normal log returns with a finite variance [8]. The MMAR nests the fractionally-integrated ARFIMA(0, d, 0) model, which allows for long memory in returns, and can also accommodate long memory in volatility associated with multifractality in the trading process. Long memory in returns does not necessarily imply multifractality, and vice versa. Thus, the MMAR is able to replicate the pricing behaviour of many types of financial assets.

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The MMAR describes a continuous-time process, constructed by compounding fractional Brownian motion (FBM) with a random, multifractal time-deformation process. The time-deformation process allows for volatility clustering in log returns measured at any time scale, and for long memory in volatility.

The property of multifractality is identified empirically through estimation of the moment scaling function, $E(|\Delta^{(n)} p_t|^q)$, where p_t is log price, and $\Delta^{(n)} p_t = p_t - p_{t-n}$ is the log return measured over the time scale n, over a range of values for q. Multi-scaling behaviour implies different exponents characterize the variation of different q-moments of the unconditional distribution of returns as the time scale n changes. The scaling function captures the intuition that different economic factors, such as technological shocks, business cycles and liquidity shocks, have different time scales, and that the volatility arising from shocks associated with different factors may have different degrees of persistence [8,9]. The MMAR allows for a wide range of behaviour in the tails of the unconditional distribution of returns, including fat tails at high frequencies [10]. Consistent with many financial returns series, including exchange rate returns, the MMAR allows the tails of the unconditional distribution to become thinner as the time scale increases; but the distribution need not converge to a normal distribution at the lowest frequencies [9].

Previous empirical studies employ methods such as Multi-Fractal Detrended Fluctuation Analysis [5,11–14], Wavelet Transform Modulus Maxima [11] or a Generalized Hurst Exponent approach [15] to detect multifractality. In this study, we develop statistical testing criteria for jointly-estimated long memory and multifractality parameters, based on a conventional hypothesis-testing methodology. In so doing, we facilitate comparisons between processes that are described by the MMAR, and processes that are characterized by model specifications nested within the MMAR. The latter include NIID (normal, independent and identically distributed) returns, and long-range dependent returns generated from a log price series characterized by FBM. Monte Carlo simulations are used to generate critical values for the relevant tests.

The methods are illustrated by fitting the MMAR to the daily log returns series for the exchange rates of 12 currencies against the US dollar for the period 1993–2012. Prior literature indicates that key empirical features of exchange rate returns data, including long memory in volatility, multi-scaling behaviour, and fat tails, can be represented parsimoniously using a multifractal framework [8,16–19]. Our results suggest that the data can be characterized by a variant of the MMAR that is constructed by compounding NIID returns with a multifractal time-deformation process; but the more general formulation, in which FBM is compounded with a multifractal time-deformation process, is not supported.[1]

This paper proceeds as follows. The next section describes the MMAR. Section 3 provides an explanation of how the main parameters of the MMAR can be estimated. Section 4 introduces a joint hypothesis testing framework for long memory and multifractality and Section 5 reports the results of tests based on this method as applied to foreign exchange rates data. Section 6 concludes the paper.

2. The multifractal model of asset returns

According to the MMAR, $p_t = B_H[\theta(t)]$, where $B_H[]$ denotes FBM, and $\theta(t)$ denotes a time-deformation process. In order to construct $\theta(t)$, consider first the case $T = 2^K$ for some integer value of K. The specification of $\theta(t)$ is

$$\Delta\theta(t) = \theta(t) - \theta(t-1) = T\Omega^{-1} \prod_{k=1}^{K} m[\eta_t(k)], \quad \text{where } \Omega = \sum_{\tau=1}^{T} \prod_{k=1}^{K} m[\eta_\tau(k)] \tag{1}$$

where $\eta_t(k) = h$ if $2^{-k}(h-1)T + 1 \le t \le 2^{-k}hT$ for $k = 1 \dots K$, $h = 1 \dots k$, and $t = 1 \dots T$; and the multiplier $m[\eta_t(k)]$ is assumed to be drawn randomly from a lognormal distribution with mean λ and variance $\sigma^2 = 2(\lambda - 1)/\ln 2$ for$\lambda > 1$.

In the case $T \ne 2^K$ for any integer value of K, the following adjustments are required. Let K^* denote the minimum value of K such that $T < 2^K$. The multipliers $m[\eta_s(k)]$ are constructed in accordance with the procedure described above, for $k = 1 \dots K^*$ and $s = 1 \dots 2^{K*}$.

$$\Delta\theta(t) = T\Omega^{-1} \prod_{k=1}^{K} m[\eta_{r+t}(k)], \quad \text{where } \Omega = \sum_{\tau=r+1}^{r+T} \prod_{k=1}^{K} m[\eta_{r+\tau}(k)] \tag{2}$$

where r is a randomly drawn integer, distributed uniformly over the interval $(0, K^* - T)$.

Let $u_t \sim N(0, \Delta\theta(t))$. For $\lambda = 1$, $\Delta\theta(t) = 1$ for all t. u_t is homoscedastic and there is no multifractality in u_t. For $\lambda > 1$, u_t is heteroscedastic and there is multifractality in u_t. Combining the multifractal time-deformation process with FBM such that $p_t = B_H[\theta(t)]$, the data generating process for $\Delta^{(1)} p_t$ is

$$(1 - L)^d \Delta^{(1)} p_t = u_t \tag{3}$$

where L denotes the lag operator $L^s \Delta^{(1)} p_t = \Delta^{(1)} p_{t-s}$ for $s = 0, 1, 2, \dots$.

[1] A methodological approach that involves fitting a multifractal model to data, and drawing inferences from the fitted parameter values, has a longstanding tradition in the multifractality literature, extending back to Ref. [20]. In this study and elsewhere, multifractality is the (model-dependent) alternative hypothesis against which null hypotheses favouring a more parsimonious model specification are tested.