Physica A 447 (2016) 422–433

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Optimizing the controllability of arbitrary networks with genetic algorithm

Xin-Feng Li, Zhe-Ming Lu *

School of Aeronautics and Astronautics, Zhejiang University, Hangzhou, 310027, China

HIGHLIGHTS

- The problem of optimizing controllability of arbitrary networks is studied.
- An efficient genetic algorithm oriented controllability optimization framework is proposed.
- The evolution of network topology is captured.
- How a network's structure affects its controllability is explored.

ARTICLE INFO

Article history:
Received 27 May 2015
Received in revised form 4 December 2015
Available online 29 December 2015

Keywords:
Complex networks
Controllability optimization
Optimal topology
Genetic algorithm

ABSTRACT

Recently, as the controllability of complex networks attracts much attention, how to optimize networks' controllability has become a common and urgent problem. In this paper, we develop an efficient genetic algorithm oriented optimization tool to optimize the controllability of arbitrary networks consisting of both state nodes and control nodes under Popov–Belevitch–Hautus rank condition. The experimental results on a number of benchmark networks show the effectiveness of this method and the evolution of network topology is captured. Furthermore, we explore how network structure affects its controllability and find that the sparser a network is, the more control nodes are needed to control it and the larger the differences between node degrees, the more control nodes are needed to achieve the full control. Our framework provides an alternative to controllability optimization and can be applied to arbitrary networks without any limitations.

1. Introduction

Complex networks are ubiquitous in nature and society [1–4], describing various systems such as food webs [5], biochemical networks [6,7], social networks [8], citation networks [9,10], and so on. Discovering their behind principles will enrich our understanding of complex natural and human systems. Previous research has mostly been focused on modeling [11,12], measuring [4], synchronization [13], navigation and search [14], community detection [15–17], epidemic spreading [18–20], robustness [21,22], and so on.

However, the ultimate proof of our understanding of complex systems is reflected in our ability to control them [23], which means injecting the external signals to some suitable nodes (called driver nodes) so that the system can be driven from any initial state to any desired final state within finite time. Considerable efforts [24–29] have been devoted to uncover the relationship between network topology and its controllability. The ground-breaking contribution was made by Liu et al. [23] who developed a minimum input theory to efficiently characterize the structural controllability of directed

* Corresponding author. Tel.: +86 13868074895.
E-mail address: zheminglu@zju.edu.cn (Z.-M. Lu).

http://dx.doi.org/10.1016/j.physa.2015.12.007
0378-4371/

networks, allowing the minimum number of driver nodes (N_D) to be identified to achieve full control. Their simulation results show that N_D is mainly determined by networks' degree distribution and the sparse inhomogeneous networks are the most difficult to control. The structural controllability framework opens a great avalanche of research on network controllability, several basic issues have been carefully addressed, such as linear edge dynamics [30], lower and upper bounds of energy required for control [31], control centrality [32], robustness of controllability [33], and so on.

In addition to the above issues, how to optimize the controllability of networks has become a common and urgent problem. Wang et al. [34] first investigate this problem and propose a perturbation strategy based on adding the least number of edges (only $N_D - 1$ edges) for connecting the separated control paths in a proper sequence, which brings the network under control with only one driver node. Despite its excellent performance, adding edges disturbs the network topology and is not practical and economic in practice. Thus Hou et al. [35] propose an efficient heuristic approximation algorithm to assign edge direction based on node residual degree, which neither changes the original structure of network nor pays additional cost for adding links. However, this algorithm cannot guarantee the optimal orientation of controllability for the given network topology because of its random feature of selecting nodes or edges. To find the optimal edge orientation, Xiao et al. [36] systematically study the edge orientation of optimal controllability problem and transfer this problem to find the maximum independent set of the constructed switching network. Besides, Iudice et al. [37] present an exact method to optimize the network controllability based on the solution of an integer linear programming (ILP) in which some physical and economic constraints are considered. All the above studies assume that the link weights of networks are precisely unknown and the networks taken into consideration only consist of state nodes. However, most real-world complex systems physically consist of both state nodes and control nodes and the linking strength between nodes could be reasonably determined by principle knowledge. Therefore, Ding et al. [38] propose the problem of optimizing the controllability of networks consisting of both "state nodes" and "control node", and develop an extremal optimization oriented heuristic tool to find the optimal network topology with the minimum number of control nodes while maintaining the network's full controllability under Kalman rank condition [39].

Although Ding et al.'s work [38] offers a general tool to optimize the controllability of networks consisting of both state nodes and control nodes, their method is only valid for directed networks. In order to speed up the algorithm, the authors take structural controllability framework [23] as a pre-test. However, the structural controllability framework is only applicable to directed networks characterized by structural matrices in which all links are represented by independent free parameters [40]. For undirected networks or networks where exact link weights are available, the assumption of structural matrix is violated, thus this method fails.

To overcome this limitation, in this paper we develop an efficient genetic algorithm oriented heuristic tool to optimize the controllability of networks consisting of both state nodes and control nodes with arbitrary structures and link weights on the basis of Popov–Belevitch–Hautus (PBH) rank condition [41], which is equivalent to Kalman rank condition [39]. The experimental results on a number of benchmark networks show the effectiveness of this method and the evolution process of network topology is captured. Furthermore, we investigate how network's structural properties affect its controllability.

The rest of the paper is organized as follows. In Section 2, we introduce the preliminary results of network controllability. In Section 3, we give the problem formulation. In Section 4, we detail the genetic algorithm oriented optimization framework. In Section 5, simulation results and discussions are presented. Finally, Section 6 concludes the paper.

2. Preliminaries

Consider a network of N nodes described by the following ordinary differential equation [23]:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_N)^T$ captures the states of N nodes, $A \in \mathbb{R}^{N\times N}$ denotes the coupling matrix of the system, in which a_{ij} represents the weight of directed link from node x_j to x_i (for undirected networks, $a_{ij} = a_{ji}$). $\mathbf{u} = (u_1, u_2, \ldots, u_P)^T$ is the input or control vector of P controllers, and $B \in \mathbb{R}^{N\times P}$ is the input matrix, in which b_{ij} represents the weight of directed link from control node u_j to state node x_i.

The system described by Eq. (1) is said controllable if it can be driven from any initial state to any desired final state within finite time. According to the classic Kalman rank condition [39], that is possible if and only if the $N \times NP$ controllability matrix

$$C = (B, AB, A^2B, \ldots, A^{N-1}B) \tag{2}$$

has full rank, that is

$$\mathrm{rank}(C) = N. \tag{3}$$

The standard approach to the controllability problem is to find a suitable matrix B consisting of the minimum number of columns so as to satisfy the Kalman rank condition. However, a practical difficulty is that there are 2^N possible combinations of placing controllers and the weight of links are often not known precisely. Therefore, the concept of structural controllability [42] is introduced to overcome this difficulty [23].

Eq. (1) is structurally controllable if it is possible to assign values to the nonzero entries of the matrices A and B such that (3) is verified. Note that a structurally controllable system is controllable for almost all weight combinations, except for