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Finding modules and hierarchy in weighted financial network using transfer entropy

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HIGHLIGHTS

- We construct the information transfer network based on the transfer entropy.
- We analyze the modular structure with various time resolutions.
- We compare the results with modular structure obtained from the cross correlations.
- We show that the transfer entropy provides a better modular structure with higher value of modularity.

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ABSTRACT

We study the modular structure of financial network based on the transfer entropy (TE). From the comparison with the obtained modular structure using the cross-correlation (CC), we find that TE and CC both provide well organized modular structure and the hierarchical relationship between each industrial group when the time scale of the measurement is less than one month. However, when the time scale of the measurement becomes larger than one month, we find that the modular structure from CC cannot correctly reflect the known industrial classification and their hierarchy. In addition the measured maximum modularity, Q_{max}, for TE is always larger than that for CC, which indicates that TE is a better weight measure than CC for the system with asymmetric relationship.

1. Introduction

Recent development of network science has been provided very useful and comprehensive framework to investigate the interwoven connectivity patterns observed in a wide range of scientific disciplines from physics to biology and economics [1]. In many real networks such as social networks [2], brain networks [3], protein-interaction network [4], each node belongs to a module or community. The module is a group of nodes which form a tightly knit group with high density of within-group edges and a lower density of between-group edges [5]. Such modules or communities are mesoscale building blocks of complex networks, because they usually correspond to the fundamental functional blocks in a network. Therefore, classifying modules in a network has been a fundamental problem to understand the origin of the specific topological, functional, and dynamical properties of a network.

Most studies on the modular structure of a given network have been focused on the finding of an efficient algorithm from a given topological information. Examples include the modularity maximization [5], clique percolation [6], and spectral analysis of the non-backtracking matrix [7]. Due to the inherent complexity, developing a more efficient model algorithm is still an open problem in network science. Besides finding the efficient algorithm, uncovering the relationship between

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the given modules is also an important quest to understand the organization of complex systems. Especially, the hierarchy between the modules in a network is one of the important and pervasive features of the organization of natural and artificial systems out of equilibrium [4,8–10]. Thus, finding the hierarchical relationship between modules potentially provides significant insight into the central aspects governing the physical properties of networks and their functionality.

There is an additional difficulty in finding modules and their relationship in many real networks. Many real networks are well described by the weighted networks in which each link is associated with a weight [11]. Examples of weighted networks include scientist collaboration network and airport network [12]. Even though the topological definition of the modularity for the weighted networks can be easily extended from that for the unweighted network [13], finding a good measure for weight of each link is not a trivial problem. Therefore, in order to understand the dynamical and topological properties of such weighted networks, it is very important to find more informative weight measure for network analysis of various systems.

One widely used measure for the weight is the cross correlation (CC), which is usually assumed to be symmetric [14–21]. For example, in financial system, Mantegna introduced a method to find a hierarchical arrangement of the stocks based on the CC of asset returns [14]. By defining an appropriate metric, they constructed the minimum spanning tree (MST) from the fully connected weighted graph and identified the clusters of companies. More recently, the study on the time dependent properties of CC distribution and the dynamic asset tree showed quantitative differences between the crash and the normal periods [21].

However, in many real complex systems, the relationship between each unit is not necessarily symmetric. One of important factors for such asymmetry is the causality. The causality in complex system was usually measured by the lagged CC [22], Granger causality [23], and the time-delayed mutual information [24]. The lagged CC is intuitive and simple measure for the asymmetric interaction between each unit in complex systems. By using the lagged CC, Kullmann et al. constructed a weighted directed network and quantitatively showed that there is some pulling effect between companies in financial system [22]. The causality network between global market indices based on the Granger causality was also studied [25]. Time-delayed mutual information provides more general and intuitive measure for the dependence between random variables. But it was recently shown that the mutual information does not explicitly distinguish the actually exchanged information due to a common history or input signal [26]. As an alternative measure of the information transfer, the transfer entropy (TE) was introduced to exclude such undesired influences [26]. In financial systems, such as global market indices, the causality measured by TE between the market indices is well represented by the weighted directed edges [27].

In this paper, to investigate how useful TE is as a weight measure for financial system, we consider the information transfer network (ITN), in which TE is used as the weight measure, and analyze the modular structure. The modular structures of ITN are compared with those of correlation network (CN) which uses the cross correlation to determine weight between companies. From the comparison, we find that the modules of both ITN and CN are consistent with the well known industrial classification [28] when the time scale of the measurement is small. However, if the time scale becomes larger, then the modules in CN significantly deviate from the known industrial classification. In addition, the measured maximum modularity, $Q_{\max}$, of ITN is always larger than that of CN, which indicates that TE is a better weight measure than CC for the systems in which the asymmetric relationship between each unit becomes important.

2. Data set and definition of states

In order to study modular structure of the financial network and their hierarchical relationship, we use the Standard & Poor's (S&P) 100 data traded from 03/01/1962 to 03/12/2010 [29]. From the obtained time series of S&P 100 index, we first define the state, i_t, of company I at day t to calculate TE. As the simplest choice of i_t for a company I we consider the binary state, i.e. $i_t = 1\,(0)$ if $Y_I(t+\Delta t) \geq Y_I(t)$ $(Y_I(t+\Delta t) < Y_I(t))$, where $Y_I(t)$ denotes the stock price of company I at time t. Thus i_t simply represents the increase (decrease) of price if $i_t = 1$ $(i_t = 0)$.

3. Transfer entropy and cross correlation

Let i_t (j_t) be the state of company I (J) at time t. TE which represents the information flow from J to I is defined as [26]

$$T_{J\to I} = \sum p(i_{t+1}, i_t^{(k)}, j_t^{(\ell)}) \log_2 \frac{p(i_{t+1}|i_t^{(k)}, j_t^{(\ell)})}{p(i_{t+1}|i_t^{(k)})}. \tag{1}$$

Here we use the shorthand notation $i_t^{(k)} = (i_t, \ldots, i_{t-k+1})$. The sum in Eq. (1) represents the sum over all available realization of state $(i_{t+1}, i_t^{(k)}, j_t^{(\ell)})$ in a time series. $p(i_{t+1}, i_t^{(k)}, j_t^{(\ell)})$ is the joint probability that the combination of i_{t+1}, $i_t^{(k)}$ and $j_t^{(\ell)}$ has a particular value, and $p(i_{t+1}|i_t^{(k)}, j_t^{(\ell)})$ is the conditional probability that i_{t+1} has a particular value when the values of the previous samples $i_t^{(k)}$ and $j_t^{(\ell)}$ are given. k and ℓ in Eq. (1) are set as $k = \ell = 1$ [26]. In ITN the weight from a company J to I is assigned as $w_{JI} = T_{J\to I}$.

For a comparison we use CC as a weight between nodes to construct CN. CC between node I and J, G_{IJ}, is defined as [15–20]

$$G_{IJ} = \frac{\langle R_I R_J\rangle - \langle R_I\rangle \langle R_J\rangle}{\sqrt{\sigma_I^2 \sigma_J^2}}. \tag{2}$$