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# Forecasting volatility of SSEC in Chinese stock market using multifractal analysis

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**Abstract**

In this paper, taking about 7 years' high-frequency data of the Shanghai Stock Exchange Composite Index (SSEC) as an example, we propose a daily volatility measure based on the multifractal spectrum of the high-frequency price variability within a trading day. An ARFIMA model is used to depict the dynamics of this multifractal volatility (MFV) measures. The one-day ahead volatility forecasting performances of the MFV model and some other existing volatility models, such as the realized volatility model, stochastic volatility model and GARCH, are evaluated by the superior prediction ability (SPA) test. The empirical results show that under several loss functions, the MFV model obtains the best forecasting accuracy.


*Keywords:* Econophysics; Multifractal; Realized volatility; Stochastic volatility model; GARCH; Superior predictive ability

## 1. Introduction

Modeling and forecasting volatility in financial markets is a key issue in many important fields, such as derivative products pricing, portfolio allocation and risk measurement. The seminal paper of Engle [1] has paved the way for the development of a large number of so-called historical volatility models in which a time varying volatility process is extracted from financial return data. Many of these models can be regarded as variants of the generalized autoregressive conditional heteroskedasticity (GARCH) models [2]. A rival class for ARCH is associated with the stochastic volatility (SV) models [3].

Both GARCH and SV models are regularly used for the analysis of daily, weekly and monthly returns. However the recent widespread availability of intraday high-frequency prices of financial assets and the work done on them have shed new light on the concept of volatility: as a matter of fact, data sampled at regular intradaily intervals can be summarized into a measure called realized volatility (RV) which, under some assumptions, is a consistent estimator of the quadratic variation of the underlying diffusion process [4]. In principle, the volatility measures derived from high-frequency data should prove to be more accurate, hence allowing for forecast efficiency gains. Nevertheless, recently Ref. [5] shows that realized volatility is prone to all sorts of microstructure problems.

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Since the suggestion of Mandelbrot [6] that multifractal is a powerful tool for depicting volatility complexities in financial markets, much research has been done in this field. However most of these studies focus on empirical tests of multifractality in different financial data sets. So we wonder whether multifractal analysis can contribute to the measurement and forecasting accuracy of volatility in financial markets. Taking high-frequency data of SSEC index in Chinese stock market as an example, first we propose a so-called multifractal volatility (MFV) measure based on the multifractal spectrum of high-frequency price movements within one trading day. Second similar to realized volatility, we also propose an ARFIMA process to model the dynamics of MFV and use a rolling-window method to forecast the volatility of SSEC one day ahead. Finally, we use a formal test for superior prediction ability (SPA) proposed by Ref. [7] to evaluate the forecasting performance of the MFV model and compare it to other popular volatility models, such as RV, SV and GARCH models. The empirical results show that under several loss functions, i.e., mean square error adjusted for heteroskedasticity (HMSE) and mean absolute error adjusted for heteroskedasticity (HMAE), the MFV model obtains the best forecasting accuracy.

This paper is organized as follows. In the next section, we introduce the sample data and discuss how daily and intraday returns are constructed. In Section 3, we discuss how realized volatility is derived from intraday returns and the ARFIMA model for RV. In Section 4, we introduce the calculation of the multifractal volatility measure from the multifractal spectrum of high-frequency price movements within one trading day. In Section 5, the historical volatility models are briefly described. The out-of-sample forecasting methodology and SPA test are discussed in Section 6, and in Section 7, the estimation and forecasting results are presented. Section 8 summarizes the conclusions.

## 2. The data, daily and intraday returns

The data for our empirical study consists of high-frequency (every 5 min) price quotes of the Shanghai Stock Exchange Composite Index (SSEC), the most important stock index in the Chinese stock market, during the period from 19 January 1999 to 30 December 2005, which contains totally $N = 1670$ trading days. The Shanghai Stock Exchange is open from 9:30 a.m. to 11:30 a.m. and then from 1:00 p.m. to 3:00 p.m., so there are 4 trading hours in a trading day, and there are 48 quotes (per 5 min) of index in a day (excluding the open price). The 5-min price quotes are denoted as $I_{t,d}$, $t = 1, 2, \ldots, N$ and $d = 0, 1, 2, \ldots, 48$. $I_{t,0}$ denotes the open price on day $t$ and $I_{t,48}$ the close price quote. Here the daily return $R_t$ is defined as

$$R_t = 100(\ln I_{t,48} - \ln I_{t-1,48}), \tag{1}$$

and the intraday high-frequency return $R_{t,d}$ is defined as

$$R_{t,d} = 100(\ln I_{t,d} - \ln I_{t,d-1}), \quad d = 1, 2, \ldots, 48. \tag{2}$$

## 3. Realized volatility and the ARFIMA model

It is generally accepted that squared daily returns provide a poor approximation of actual daily volatility. Ref. [4] first points out that more accurate estimates can be obtained by summing squared intraday returns. If we would apply their method directly in this paper, we would define realized volatility as

$$RV_t' = \sum_{d=1}^{48} R_{t,d}^2. \tag{3}$$

However, this definition ignores the information obtained in the overnight returns. In order to account for this problem, Ref. [8] suggests scaling the realized volatility in this way:

$$RV_t = \gamma RV_t', \tag{4}$$

where the so-called scale parameter $\gamma$ is defined as

$$\gamma = \frac{N^{-1} \sum_{t=1}^{N} R_t^2}{N^{-1} \sum_{t=1}^{N} RV_t'}. \tag{5}$$