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On animal spirits and economic decisions: Value-at-Risk and Value-within-Reach as measures of risk and return

Domingo Castelo Joaquin

Department of Finance, Insurance, and Law, Illinois State University, Normal, IL 61790, United States

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ABSTRACT

Let us suppose that presently unimagined is possible, that "the unexpected may happen" (Marshall (1920). *Principles of economics*, McMillan, London, p. 347). Then "human decisions affecting the future, whether personal, political or economic, cannot depend on strict mathematical expectation since the basis for making such calculations does not exist" (Keynes (1936). The general theory of employment, interest and money, Harcourt Brace, New York, NY, pp. 162–163) and "individual initiative will only be adequate when reasonable calculation is supplemented and supported by animal spirits" (Keynes (1936). *The general theory of employment, interest and money*, Harcourt Brace, New York, NY, p. 162)—by "a spontaneous urge to action rather than inaction" (Keynes (1936). *The general theory of employment, interest and money*, Harcourt Brace, New York, NY, p. 161). It is intended here to examine an investment's Value-at-Risk as a reasonable calculation of the worst threat an action appears to make possible, and its return counterpart, referred to as the investment's Value-within-Reach, as a reasonable calculation of the best hope an action appears to offer. In exploring the extension of the Value-at-Risk approach from applications to investments in financial assets to applications to investments in real assets, the properties of Value-at-Risk as a risk measure are reviewed. Recognizing that Value-at-Risk focuses exclusively on downside risk, a complementary set of properties is specified which is shown to be necessary and sufficient for the acceptance of Value-within-Reach as a measure of return. This note concludes with remarks on a distribution's focus-values, consisting of the distribution's Value-at-Risk and Value-within-Reach, as reasonable calculation of a course of action's risk and return.

1. Introduction

In a Department of Defense Briefing on February 12, 2002, Secretary Rumsfeld described the normal state of knowledge or, rather, un-knowledge that one normally operates in as follows: "Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns—the ones we don't know we don't know." (Rumsfeld, 2002). When the presently unimagined is possible, when "the unexpected may happen" (Marshall, 1920, p. 347), then "human decisions affecting the future, whether personal, political or economic, cannot depend on strict mathematical expectation since the basis for making such calculations does not exist" (Keynes, 1936, pp. 162–163) and "individual initiative will only be adequate when reasonable calculation is supplemented and supported by animal spirits" (Keynes, 1936, p. 162)—by "a spontaneous urge to action rather than inaction" (Keynes, 1936, p. 161). Within this context, it is intended here to examine an investment's Value-at-Risk as a reasonable calculation of the worst threat an action appears to make possible, and its return counterpart, referred to as the investment's Value-within-Reach, as a reasonable calculation of the best hope an action appears to offer. In exploring the extension of the Value-at-Risk approach from applications to investments in financial assets to applications to investments in real assets, the properties of Value-at-Risk as a risk measure are reviewed. Recognizing that Value-at-Risk focuses exclusively on downside risk, a complementary set of properties is specified which is shown to be necessary and sufficient for the acceptance of Value-within-Reach as a measure of return. This note concludes with remarks on a distribution's focus-values, consisting of the distribution's Value-at-Risk and Value-within-Reach, as reasonable calculation of a course of action's risk and return.

E-mail address: dcjoaqu@ilstu.edu

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1.1. Notation

$(\Omega, \Im, P)$	Probability space
$X, Y \in G$	Real-valued random variables on $(\Omega, \Im, P)$
$\rho(*; \alpha): G \to R$	Risk measure on G with parameter $\alpha \in (0,1)$
$q(X; \alpha) = \inf\{x \in R: P(X \le x) \ge \alpha\}$	α-Quantile of X; $\alpha \in (0,1)$ and $\inf \emptyset = \infty$
$\mathrm{VaR}(X; \alpha) = -q(X; \alpha)$	Value-at-Risk of X at confidence level $\alpha \in (0,1)$
$\lambda(*; \theta): G \to R$	Return measure on G with parameter $\theta \in (0,1)$
$Q(X; \theta) = \inf\{x \in R: P(X \ge x) \le \theta\}$	Upper α-quantile of X; $\theta \in (0,1)$ and $\inf \emptyset = \infty$
$\mathrm{VwR}(X; \theta) = Q(X; \theta)$	Value-within-Reach of X at confidence level $\theta \in (0,1)$

2. Properties of a return measure

In an earlier note (Joaquin, 2009), I showed that, for any given confidence level $\alpha \in (0,1)$, a risk measure $\rho(*; \alpha)$ is consistent with Value-at-Risk (VaR) ordering if and only if the risk measure satisfies the following properties:

Property 1A. *Weak dominance:* If the probability of a worse outcome than any given value is greater for X than for Y, then X is at least as risky as Y[1].

Property 1B. *Only sufficiently likely threats matter:* Risk cannot be reduced by redistributing probability mass in the lower α-tail of a distribution[2].

Property 1C. *Focus on the downside risk:* Risk cannot be increased by redistributing the probability mass in the upper $1-\alpha$ tail of a distribution[3].

Property 1C requires exclusive focus on downside risk, which is not a surprising requirement for a risk measure. At the same time, Property 1B allows for and, in fact, prescribes insensitivity to the worse of the downside. The reasonableness of this requirement depends in large measure on the decision maker's confidence on the completeness of his or her expectations[4]. The insensitivity to tail transformations required by Properties 1B and 1C represents a way of dealing with the possibility of change in own expectations. Finally, the dominance condition in Property 1A is of the weak form to retain consistency with Properties 1B and 1C[5].

To be useful for capital budgeting applications, Value-at-Risk needs to be complemented by a measure of upside risk. There are three properties of a return measure the acceptance of all of which implies the acceptance of what is referred to as Value-within-Reach (VwR) as a return measure; and the rejection of any one of which implies the rejection of the Value-within-Reach (VwR) as a return measure[6]. These properties are similar in spirit to the properties of Value-at-Risk, but applied to upside risk.

Property 2A. *Weak dominance:* If, for any given value, the probability of achieving at least as good an outcome for X is at least as high as the probability for Y, then X is at least as attractive as Y.

$$\text{If } P(X \ge t) \ge P(Y \ge t) \quad \text{for all } t \in R, \quad \text{then } \lambda(X;\theta) \ge \lambda(Y;\theta). \tag{1}$$

Property 2B. *Only sufficiently likely prospects matter:* Attractiveness cannot be improved by redistributing the probability mass in the upper θ-tail of a distribution, for any given $\theta \in (0,1)$. If Y can be derived from X by redistributing the probability mass in the upper θ-tail of the distribution of X, then Y cannot be more attractive than X[7].

$$\text{If } P(X > t) = P(Y > t) \quad \text{for all } t < Q(X;\theta), \quad \text{then } \lambda(X;\theta) \ge \lambda(Y;\theta). \tag{2}$$

Property 2C. *Focus on the upside risk:* Attractiveness cannot be improved by redistributing probability mass in the lower $(1-\theta)$ tail of a distribution, for any given $\theta \in (0,1)$. If Y can be derived from X by redistributing the probability mass in the lower $(1-\theta)$-quantile of the distribution of X, then Y cannot be more attractive than X[8].

$$\text{If } P(X > t) = P(Y > t) \quad \text{for all } t > Q(X;\theta), \quad \text{then } \lambda(X;\theta) \ge \lambda(Y;\theta). \tag{3}$$

Together, Properties *B* and *C* link the return measure to the distribution's upper θ-quantile and, therefore, to its Value-within-Reach, VwR.

3. A characterization of Value-within-Reach (VwR) as a return measure

Lemma. For a fixed $\theta \in (0,1)$, a return measure $\lambda(*; \theta)$ satisfies Properties 2A–2C if and only if $\lambda(X; \theta) \le \lambda(Y: \theta)$ if $\mathrm{VwR}(X; \theta) \le \mathrm{VwR}(Y; \theta)$.

Proof. *Necessity.* For fixed $\theta \in (0,1)$, suppose that a return measure $\lambda(*; \theta)$ satisfies Properties 2A–2C. Let X and Y be random variables on $(\Omega, \Im, P)$ such that $\mathrm{VwR}(X; \theta) \le \mathrm{VwR}(Y; \theta)$. Want to show that $\lambda(X; \theta) \le \lambda(Y: \theta)$. Note that $Q(X; \theta) \le Q(Y; \theta)$ since $\mathrm{VwR}(*;\theta) = Q(*; \theta)$.

Define Y^* as follows:

$$Y^*(\omega) = \begin{cases} Q(Y;\theta) & \text{for } \omega \text{ in } \{\omega \in \Omega : Y(\omega) \ge Q(Y;\theta)\} \\ Y(\omega) & \text{for } \omega \text{ in } \{\omega \in \Omega : Y(\omega) < Q(Y;\theta)\} \end{cases} \tag{4}$$

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[1] If $P(X < t) \ge P(Y < t)$ for all $t \in R$, then $\rho(X; \alpha) \ge \rho(Y; \alpha)$.

[2] If $P(X < t) = P(Y < t)$ for all $t > q(X; \alpha)$, then $\rho(X; \alpha) \le \rho(Y; \alpha)$.

[3] If $P(X < t) = P(Y < t)$ for all $t \le q(X; \alpha)$, then $\rho(X; \alpha) \ge \rho(Y; \alpha)$.

[4] The acceptability of this property may also depend on how bad the worse can get. An alternative model might be preferred in the presence of catastrophic risk. See, for example, Chichilnisky (2000)and Acerbi and Tasche (2002).

[5] For example, it is possible that $P(X \le t) \ge P(Y \le t)$ for all $t \in R$ and $P(X \le t) > P(Y \le t)$ for some t and yet Property 1A would allow having only $\rho(X; \alpha) = \rho(Y; \alpha)$. A stricter version of the dominance property, requiring $\rho(X; \alpha) > \rho(Y; \alpha)$ in this case, would not be consistent with Properties B and C.

[6] For any $\theta \in (0,1)$, if the return measure of X is higher than that of Y, we shall say that X is more attractive than Y, and write $\lambda(X; \theta) > \lambda(Y;\theta)$. The relations "less attractive" and "equally attractive" are similarly defined. The term "return measure" will be used interchangeably with the term "attractiveness measure."

[7] For example, suppose $\theta = 0.50$. Let X be a discrete random variable with distribution $P(X = -1000) = 0.10$, $P(X = 500) = 0.40$, and $P(X = 1000) = 0.50$. Let Y be derived from X by redistributing the probability mass in the upper θ-quantile of the distribution of X, such that Y has a distribution $P(Y = -1000) = 0.10$, $P(Y = 500) = 0.40$, and $P(Y = 1000) = 0.20$, and $P(Y = 1500) = 0.30$. Then, Property 2B asserts that Y cannot be more attractive than X.

[8] For example, suppose $\theta = 0.50$. Let X be a discrete random variable with distribution $P(X = -1000) = 0.10$, $P(X = 500) = 0.40$, and $P(X = 1000) = 0.50$. Let Y be derived from X by redistributing the probability mass in the lower $1-\theta$ quantile of the distribution of X, such that Y has a distribution $P(Y = -1000) = 0.10$, $P(Y = 500) = 0.20$, $P(Y = 600) = 0.20$, and $P(Y = 1000) = 0.50$. Then, Property 2C asserts that Y cannot be more attractive than X.