NDT and E International 100 (2018) 101–107

Contents lists available at ScienceDirect

NDT and E International

journal homepage: www.elsevier.com/locate/ndteint

Parameter estimation from pulsed thermography data using the virtual wave concept

G. Mayr[a,*], G. Stockner[a], H. Plasser[a], G. Hendorfer[a], P. Burgholzer[b]

[a] Josef Ressel Centre for Thermal NDE of Composites, University of Applied Sciences Upper Austria, Wels, Austria
[b] RECENDT - Research Centre for Nondestructive Testing, Linz, Austria

ARTICLE INFO

Keywords:
Active thermography
Photothermal technique
Virtual wave concept
Parameter estimation
Inverse problem
Regularization

ABSTRACT

The aim of this work is the estimation of the specimen thickness from pulsed thermography data using the virtual wave concept. A virtual wave signal is calculated by applying a local transformation to the measured temperature data. This virtual wave is a solution of the wave equation, whereby for the parameter estimation also ultrasonic evaluation methods can be used, e.g. pulse-echo method for time-of-flight measurements. The time-of-flight is directly related to the distance traveled by the wave and can be used to reconstruct the position of the interface. This method yields a very good estimation of the thickness of a steel step wedge, with the advantage that the same evaluation method can be used for reflection as well as transmission measurements.

1. Introduction

In the recent years pulsed thermography has been successfully applied for non-destructive testing and evaluation (NDT&E) of material and components [1]. For depth estimation and in some cases for the characterization of the thermal resistance between the bulk material and the discontinuity usually a one-dimensional (1D) thermal model is used. The thermal signal reconstruction (TSR) - technique [2] and its combination with an early contrast approach [3] allows the estimation of the defect depth and thermal resistance [4]. For defect detection the Pulse Phase Thermography (PPT) method analyses the measured temperature data in the frequency domain [5]. Several quantitative inversion methods have been proposed to process PPT data, e.g. the blind frequency method [6]. In addition to these well-known methods, there are also many other approaches like the synthetic thermal time-of-flight (STTOF) [7], dynamic thermal tomography (DTT) [1] or identification methods based on the thermal contrast [8,9]. There are also new approaches in modulated photothermal radiometry to estimate the thermal diffusivity or sample thickness [10,11].

In recent years interest has grown in the theory and application of inverse heat transfer techniques to estimate material parameters or to reconstruct defects. These techniques involve the minimization of an objective function which contains the square of differences between the experimental and model-based temperature at each time step. The ill-posed nature of the thermal reconstruction problems leads to large excursion and oscillation in the solution if quite small errors (noise) in the measurement data occur. One approach to reduce such instabilities is to use regularization procedures [12]. These are already used for 1D thermal NDE problems, e.g. reconstruction of the thermal effusivity- [13] or thermal conductivity- [14,15] depth profiles.

In this paper we evaluate pulsed thermography data from the perspective of ultrasonic testing. The virtual wave concept allows to apply ultrasonic evaluation methods, like time-of-flight (TOF) on pulsed thermography measurements [16]. We show that by a local transformation of the measured temperature, it is possible to process the data appropriately to obtain the TOF or, equivalently the distance z. In this study we estimate the thickness of metallic steps wedges from optical-excited pulsed thermography measurements in transmission and reflection configuration.

2. Virtual wave concept

The non-stationary heat conduction process in solids is described by the heat flux proportional to the temperature gradient which leads to the heat diffusion equation

$$\left(\nabla^2 - \frac{1}{\alpha}\frac{\partial}{\partial t}\right)T(\mathbf{r}, t) = -\frac{1}{\alpha}T_0(\mathbf{r})\,\delta(t), \tag{1}$$

where $T(\mathbf{r}, t)$ is the temperature as a function of space and time and $\alpha = k/(\rho\, c_p)$ is the thermal diffusivity with the thermal conductivity k, the density ρ and the heat capacity c_p. The source term on the right side describes a impulsive thermal excitation, where $T_0(\mathbf{r})$ is the initial

* Corresponding author.
E-mail address: guenther.mayr@fh-wels.at (G. Mayr).

https://doi.org/10.1016/j.ndteint.2018.09.003
Received 18 June 2018; Received in revised form 5 September 2018; Accepted 5 September 2018
Available online 08 September 2018

temperature distribution and $\delta(t)$ the temporal Dirac delta function.Ultrasound waves can be modeled using the wave equation, where the acoustic pressure $p(\mathbf{r}, t)$ is described as a function of space $\mathbf{r}$ and time t:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p(\mathbf{r}, t) = -\frac{1}{c^2}\frac{\partial}{\partial t}p_0(\mathbf{r})\,\delta(t), \tag{2}$$

where p_0 is the initial pressure just after the Dirac-like excitation impulse and c is the speed of sound in the investigated media.Based on the concept of virtual waves [16], we introduce the virtual wave $T_{\text{virt}}(\mathbf{r}, t)$. This virtual wave is defined such that the wave equation (Eq. (2)) is valid with the initial temperature distribution and an arbitrarily chosen c:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)T_{\text{virt}}(\mathbf{r}, t) = -\frac{1}{c^2}\frac{\partial}{\partial t}T_0(\mathbf{r})\,\delta(t). \tag{3}$$

The connection between the temperature signal $T(\mathbf{r}, t)$ and the virtual wave signal $T_{\text{virt}}(\mathbf{r}, t)$ at the same position $\mathbf{r}$ in the time domain is a linear inverse problem, which can be formulated as Fredholm integral of the first kind in the following form [12]:

$$\int_{-\infty}^{\infty} K(t, t')\,T_{\text{virt}}(\mathbf{r}, t')\,dt' = T(\mathbf{r}, t), \tag{4}$$

where the right-hand side T and the kernel K are in principal known functions, while T_{virt} is the unknown solution. In our case the kernel K is given exactly by the underlying mathematical model of virtual waves, while T consists of measured temperature data. The temperature data T is only known with a certain accuracy and only in a finite set of time steps, which depends on performance of the infrared camera. The kernel K is given as

$$K(t, t') \equiv \frac{c}{\sqrt{\pi\,\alpha\,t}}\exp\left(-\frac{c^2 t'^2}{4\,\alpha\,t}\right)\text{for} \quad t > 0. \tag{5}$$

The aim is to find the virtual wave field T_{virt} from the measured temperature field T and the corresponding model K.

3. Inverse heat conduction problem

The dependent variable of the estimated parameter, in our case the temperature $T(\mathbf{r}, t)$, might be measured with infrared thermography at one or two planes of a plate: $z = [0,\ L]$, where L is the thickness. The temperature is recorded at many discrete times N_t after the thermal stimulation. The discrete time is given by $t_k = k\,\Delta_t$, where Δ_t is the step width of the time discretization and $k = 1, \ldots, N_t$ the number of time steps.

3.1. Discretization and regularization

The main difficulties in the solution of inverse heat conduction problems is that the ill-posed nature leads to a high sensitivity of the solution in dependence of discretization and measurement errors, especially when more than one parameter needs to be determined. The Fredholm integral (Eq. (4)) is usually solved numerically by obtaining a discrete approximation of the kernel (Eq. (5)). The measured temperature T_k at discrete time steps k is given by

$$\sum_{j=1}^{N} K_{kj}\,T_{\text{virt},j} = T_k, \tag{6}$$

where K_{kj} is the discrete kernel at the time steps j and k, respectively. Herein, the integration operator was replaced by the summation operator and dt' by $\Delta_{t'}$. This results in a discrete kernel in the following manner:

$$K_{kj} = \frac{\tilde{c}}{\sqrt{\pi\,\Delta_{Fo}\,k}}\exp\left(-\frac{\tilde{c}^2\,(j-1)^2}{4\,\Delta_{Fo}\,k}\right), \tag{7}$$

where $t'_j = (j-1)\,\Delta_{t'}$ is the virtual time at the j-th point with the step width of discretization $\Delta_{t'}$. For the reconstruction of the virtual wave $T_{\text{virt},j}$, we consider a one-dimensional slab at $z = 0$ with the thickness L. The z-axis is divided into N_z equally spaced elements. The width of each element is denoted as Δ_z and each element is located at $z_n = n\,\Delta_z$ with $n = 0, \ldots, N_z - 1$. The locations z_n are referred as grid points. The Fourier number Fo characterizes the transient heat conduction rate to the storage rate. For a discrete time and space step the discrete Fourier number Fo is calculated as $\Delta_{Fo} = \alpha\,(\Delta_t/\Delta_z^2)$. The dimensionless virtual speed of sound $\tilde{c} = c\,(\Delta_z/\Delta_{t'})^{-1}$ is defined as the relation of the virtual speed of sound c and spatially and temporal discretization $(\Delta_z/\Delta_{t'})$.The Fredholm integral (Eq. (4)) can be also written as matrix equation in the form of

$$\mathbf{K}\ \mathbf{T}_{\text{virt}} = \mathbf{T}, \tag{8}$$

where $\mathbf{T}_{virt}$ and $\mathbf{T}$ are the column vectors of the virtual wave and the measured temperature signal at discrete time steps. The kernel $\mathbf{K}$ ($M \times N$ matrix) can be calculated for this time steps using Eq. (7).The objective of the inverse problem is to determine the virtual wave field that depends on the (noisy) temperature measurements of the output. The Singular Value Decomposition (SVD) allows the analysis of the ill-conditioned inverse problem. The matrix $\mathbf{K}$ has a deficient numerical rank which results in a high condition number. The condition number C is defined as the ratio between the largest and the smallest singular values: $C = \mu_1/\mu_N$. It measures the solution's sensitivity to rounding and measurement errors.The matrix $\mathbf{K}$ can be inverted only with appropriate regularization, that means additional information can stabilize the solution. To treat the problem we use a direct numerical regularization method - the Truncated Singular Value Decomposition (T-SVD) [12]. We consider the matrix $\mathbf{K}$ as a noisy representation of a mathematically rank-deficient matrix $\mathbf{K}_{\text{trunc}}$. The matrix $\mathbf{K}$ is replaced by a truncated matrix $\mathbf{K}_{\text{trunc}}$, where the smallest non-zero singular values $\mu_{i_{\text{trunc}}+1}, \ldots, \mu_N$ are substituted with exact zero, where i_{trunc} is the truncation index. The T-SVD of $\mathbf{K}$ is defined as the truncated matrix

$$\mathbf{K}_{\text{trunc}} \equiv \mathbf{U}\boldsymbol{\Sigma}_{i_{\text{trunc}}}\mathbf{V}^T = \sum_{i=1}^{i_{\text{trunc}}} \mathbf{v}_i\,\mu_i\,\mathbf{u}_i^T \tag{9}$$

where $\mathbf{U} = (\mathbf{u}_1, \ldots, \mathbf{u}_M) \in \mathbb{R}^{M\times M}$ and $\mathbf{V} = (\mathbf{v}_1, \ldots, \mathbf{v}_N) \in \mathbb{R}^{N\times N}$ are matrices with orthonormal vectors. In the diagonal matrix $\boldsymbol{\Sigma}_{i_{\text{trunc}}} = \text{diag}(\mu_1, \ldots, \mu_{i_{\text{trunc}}}, 0, \ldots, 0) \in \mathbb{R}^{M\times N}$ the smallest singular values $\mu_i < \mu_{i_{\text{trunc}}}$ are replaced by zeros. $\mathbf{u}_i$ and $\mathbf{v}_i$ are the columns of the matrices $\mathbf{U}$ and $\mathbf{V}$, respectively. The regularized virtual wave $T_{\text{virt,reg}}$ can be calculated with the pseudo-inverse matrix $\mathbf{K}_{\text{trunc}}^{-1}$.

$$\mathbf{T}_{\text{virt,reg}} = \mathbf{K}_{\text{trunc}}^{-1}\,\mathbf{T} = \sum_{i=1}^{i_{\text{trunc}}} \frac{\mathbf{u}_i^T\mathbf{T}}{\mu_i}\mathbf{v}_i. \tag{10}$$

3.2. Derivation of the 1D virtual wave

In the most cases, the Fredholm integral Eq. (4) cannot be solved directly because of its ill-posed nature. An exception is given, when dealing with an infinite one-dimensional body. In this case, one can obtain the solution directly from Eq. (4) using the fundamental heat conduction solution [17].

3.2.1. The fundamental solution of the virtual wave

The Green's function solution equation for both, temperature $T(z, t)$ and virtual wave $T_{\text{virt}}(z, t')$, appropriate for an initial condition $T(z, t = 0) = F(z)$ and infinite and constant-property body, can be written as [18]:

$$T_{Z00}(z, t) = \int_{\hat{z}=0}^{\infty} G_{Z00}(z, t|\hat{z}, 0)F(\hat{z})d\hat{z} \tag{11}$$

$$T_{Z00,\text{virt}}(z, t') = \int_{\hat{z}=0}^{\infty} G_{Z00,\text{virt}}(z, t'|\hat{z}, 0)F(\hat{z})d\hat{z}, \tag{12}$$