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Global fixed-time stabilization for a class of switched nonlinear systems with general powers and its application

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ABSTRACT

This paper investigates the problem of global fixed-time stabilization for a class of uncertain switched nonlinear systems with the general powers, namely, the powers of the considered systems can be different odd rational numbers, even rational numbers or both odd and even rational numbers. By skillfully using the common Lyapunov function method and the adding a power integrator technique, a common state feedback control strategy is developed. It is proved that the proposed controller can guarantee that the state of the resulting closed-loop system converges to zero for any given fixed time under arbitrary switchings. Simulation results of the liquid-level system are provided to show the effectiveness of the proposed method.

1. Introduction

In this paper, we consider a class of switched nonlinear systems in p-normal form described by[1]

$$\begin{aligned} \dot{x}_i &= \lceil x_{i+1} \rfloor^{p_{i,\sigma(t)}} + f_{i,\sigma(t)}(\bar{x}_i), \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= \lceil u_{\sigma(t)} \rfloor^{p_{n,\sigma(t)}} + f_{n,\sigma(t)}(\bar{x}_n), \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in R^n$ is the system state; $\bar{x}_i = (x_1, \ldots, x_i)^T$, $i = 1, \ldots, n$; the function $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal which is assumed to be a piecewise continuous (from the right) function of time; m is the number of subsystems. For each $k \in M$, $u_k \in R$ is the control input of the kth subsystem. For any $k \in M$ and $i = 1, \ldots, n$, the functions $f_{i,k} : R^i \to R$ are continuous with respect to their arguments and vanish at the origin, $p_{i,k} \in R^+$ and $\lceil \cdot \rfloor^a = \text{sign}(\cdot)|\cdot|^a$, $a \in R^+$. In addition, we assume that the state of the switched system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous.

In the past decades, as an important class of hybrid dynamic systems, switched systems have attracted a great deal of attention due to their widespread applications in various fields, such as, gene regulatory networks, multi-agent systems, mechanical systems, robotic and switching power converters [1–7]. However, it has been well-recognized challenge or even impossible to study the stability analysis and global stabilization of general switched systems, because the hybrid features make that a switched system does not inherit properties of individual subsystems, for example, asymptotic stability of a switched system, may be not established for arbitrary switching signals even if all of the subsystems exhibit this property [8].

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[1] Throughout the paper, for simplicity, we will drop the argument of some functions whenever no confusion can arise; for instance, x_1 is used to denote $x_1(t)$.

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Fortunately, certain typical system structures can sometimes be utilized to obtain interesting results. Recently, a special class of switched systems, namely, switched nonlinear systems in p-normal form,[2] have been studied systematically [9–18]. For example, global stabilization of the switched nonlinear system (1) with $p_{i,k} \equiv 1$ under arbitrary switchings was studied via backstepping and common Lyapunov function method in [16] and [17], respectively. The authors in [18] further studied the case that the powers of the subsystems $p_{i,k}$'s are allowed to be different positive odd numbers, and obtained a much more general result.

However, it should be noted that the aforementioned works only consider the asymptotic behaviors of system trajectories as time goes to infinity. Nevertheless, from a practical point of view, in many applications it is more desirable that the trajectories of dynamic systems could converge to the equilibrium in finite time. As demonstrated by the authors in [19], finite-time stable systems may not only retain faster convergence, but also enjoy stronger robustness and better disturbance attention properties. In view of these superiorities, the problem of finite-time stabilization has drawn considerable attention in recent years and numerous results have been obtained, see, e.g. [20–29]. Particularly, Fu et al. in [30] recently investigated the finite-time stabilization of switched nonlinear system (1) under the assumption that $p_{i,k} \in R^{+}_{odd} := \{q \in R^{+}$: q is a ratio of odd integers$\}$, and for given numbers $p_{i,k}$, $i = 1, \ldots, n$, $k \in M$, there exist constants $0 < r_i \leq 1$ and $-1 < \tau_k < 0$ such that

$$r_1 = 1, \quad p_{i,k}r_{i+1} = r_i + \tau_k \text{ and } \ p_{i,k}r_{i+1} \leq \min\{r_1, r_2, \ldots, r_i\} \ \ i = 1, \ldots, n. \tag{2}$$

Note that the powers of subsystems of the studied system being odd rational numbers is required in this assumption. Although it is a fairly mild condition, there still exist some practical systems which do not satisfy such restriction, for example, one of the powers of the liquid-level system presented in Section 4 is $\frac{1}{2}$, which is even rather than odd rational number. In additions, it should be mentioned that the settling time functions derived in above finite-time control design rely on initial states of the studied systems, which also prohibits their practical applications to some extent, because it cannot achieve a desired performance within a preset time when the knowledge of initial conditions is unavailable in advance. As the extension, the following questions naturally arise: *Is it possible to further relax the power order restriction to cover the powers of both odd and even rational numbers? Is it possible to design a finite-time stabilizing controller whose settling time is independent of the initial conditions, i.e., fixed-time stabilizing controller? If possible, under the weaker condition how can one design a fixed-time stabilizing controller for switched nonlinear system (1)?*

By delicately combining sign function with adding a power integrator technique, we shall solve the above problems here. It is worth pointing out that neither results on the *finite-time stabilization of switched nonlinear systems whose subsystems are allowed to have the powers of even rational numbers*, nor results on the *fixed-time stabilization of switched nonlinear systems* are available in the literature. However, considering the desired property of *fixed-time stabilization* and the *generality* of studied systems, we shall focus on the study of fixed-time stabilization of switched nonlinear systems whose subsystems are allowed to have the powers of positive rational numbers. Compared to the relevant existing results in the literature, the main contributions of this paper are highlighted as follows.

- The global fixed-time stabilization problem of switched nonlinear systems, whose subsystems are allowed to have the powers of positive rational numbers, is studied for the first time.
- A weaker sufficient condition on characterizing the powers and nonlinear drifts of the switched nonlinear system for its global fixed-time stabilization is derived.
- By successfully overcoming some essential difficulties such as the weaker assumptions on the power order restriction and the system growth, and the construction of a C^1, proper and positive definite common Lyapunov function, a novel method to global fixed-time stabilization of switched nonlinear systems by state feedback is given, and leads to much more general results than the previous ones.
- An application example for liquid-level system is modeled and solved by the proposed method.

The remainder of this paper is organized as follows. Section 2 describes the problem investigated and offers some preliminary results. Section 3 presents the control design procedure and the main results, while Section 4 gives an application example to demonstrate the effectiveness of the theoretical results. Section 5 addresses some concluding remarks. The paper ends with an appendix.

2. Problem formulation and preliminaries

The objective of this paper is to design a state feedback controller for system (1) such that the origin of the closed-loop system is globally fixed-time stable under arbitrary switchings.

Towards this end, the following assumptions are needed.

Assumption 2.1. The switching signal $\sigma(t)$ has finite number of switching on every bounded time interval.

[2] The strict-feedback form is a special case of the p-normal form with $p = 1$.