The Spanish Review of Financial Economics 12 (2014) 40–45

The Spanish Review of Financial Economics

www.elsevier.es/srfe

Article

Modeling credit spreads under multifactor stochastic volatility☆

Jacinto Marabel Romo[a,b]

[a] BBVA, Vía de los Poblados s/n, 28033 Madrid, Spain
[b] Department of Management Sciences, University of Alcalá (UAH), Plaza de la Victoria 2, 28802 Alcalá de Henares, Madrid, Spain

ARTICLE INFO

Article history:
Received 10 April 2013
Accepted 26 June 2013
Available online 1 January 2014

JEL classification:
G12
G13
G31
G32
G33

Keywords:
Credit spreads
Credit rating
Stochastic volatility
Multifactor
Structural models

ABSTRACT

The empirical tests of traditional structural models of credit risk tend to indicate that such models have been unsuccessful in the modeling of credit spreads. To address these negative findings some authors introduce single-factor stochastic volatility specifications and/or jumps.

In the yield curve literature it is widely accepted that one-factor is not sufficient to capture the time variation and cross-sectional variation in the term structure. This article introduces a two-factor stochastic volatility specification within the structural model of credit risk. One of the factors determines the correlation between short-term firms' assets returns and variance, whereas the other factor determines the correlation between long-term returns and variance. The numerical tests reveal how the introduction of two volatility factors can generate a wide range of combinations associated with short-term and long-term patters corresponding to credit spreads. In this sense, multi-factor stochastic volatility specifications provide more flexibility than single-factor models to capture a wide range of shapes associated with the term structure of credit spreads consistent with the empirical evidence.

1. Introduction

Structural models of credit risk formulate explicit assumptions about the dynamics of a firm's assets, its capital structure and its debt. In this case, the default event happens if firm's assets are not sufficient to pay the debt and corporate liabilities can be considered as contingent claims on the firm's assets. The credit risk literature based on the structural approach begins with the study of Merton (1974), who applies the option pricing theory developed by Black and Scholes (1973) to model credit spreads, where the credit spread is defined as the difference between the yield of a corporate bond and the associated yield on Treasury bonds with the closest matching maturity.

The empirical tests of traditional structural models of credit risk tend to indicate that such models have been unsuccessful in the modeling of credit spreads. In particular, Jones et al. (1984) and Huang and Huang (2003), among others, show that predicted credit spreads are far below observed ones. To address these negative findings some authors, such as Zhang et al. (2009), introduce a single-factor stochastic volatility model with jumps within the Merton (1974) framework. They show that their model improves the match between predicted and observed credit spreads, especially for investment-grade companies. Unfortunately, the results are less satisfactory for low investment grade and speculative grade. In theory, jumps can help to match the observed credit spread levels for investment grade bonds and short maturities. But empirical evidence is rather inconclusive. Collin-Dufresne et al. (2003) have found that only a small fraction of observed credit spreads of aggregate portfolios can be explained by jump risk. On the other hand, Cremers et al. (2008) suggest that the addition of jumps and jump risk premia brings predicted yield spread levels much closer to observed ones.

Importantly, the study of Zhang et al. (2009) considers a single stochastic volatility factor as in Heston (1993). Within the equity option valuation context, Christoffersen et al. (2009) extend the original Heston (1993) framework to generate a two-factor stochastic volatility model built upon the square root process. This article introduces a two-factor stochastic volatility specification within the structural Merton (1974) framework. The advantage is that a two-factor model provides more flexibility to model the volatility term structure. In this sense, one of the factors determines the correlation between short-term firms' assets returns and variance, whereas the other factor determines the correlation between long-term returns and variance. Hence, the two-factor specification is able to generate more flexible credit spread term structures to match the observed term structures associated with credit spreads.

☆ The content of this paper represents the author's personal opinion and does not reflect the views of BBVA.

E-mail addresses: jacinto.marabel@bbva.com, jacinto.marabel@uah.es

2173-1268/$ – see front matter
http://dx.doi.org/10.1016/j.srfe.2013.06.002

The rest of the paper proceeds as follows. Section 2 presents the main features of the two-factor stochastic volatility specification and develops semi-closed-form solutions for the price of credit spreads and default probabilities. Section 3 provides a numerical analysis which shows that the model is able to generate credit spread term structures consistent with the empirical evidence. This section also offers a sensitivity analysis of credit spreads with respect to the model parameters. Finally, Section 4 offers concluding remarks.

2. A structural model of credit risk under a two-factor stochastic volatility framework

Let us assume the same market framework as in Merton (1974). Within this framework, firms issue a zero coupon bond with a promised payment B at maturity $t=T$. In this case, default occurs only at maturity with debt face value as default boundary. In the event of default, the absolute priority rule prevails.

Let $\left\{A_t \in \mathbb{R}\, t\geq 0\right\}$ be the price process of the firm's assets and let us denote by $Y_t = \ln A_t$ the log-return vector. For simplicity, I assume that the continuously compounded risk-free rate r and asset payout ratio q are constant. Let Θ denote the probability measure defined on a probability space $\left(\Lambda, F, \Theta\right)$ such that asset prices expressed in terms of the current account are martingales. We denote this probability measure as the risk-neutral measure. As in Christoffersen et al. (2009), I consider a two-factor specification for the variance process and I assume the following dynamics for the return process Y_t under Θ:

$$dY_t = \left[r - q - \frac{1}{2}\sum_{i=1}^{2} v_{it}\right] dt + \sum_{i=1}^{2}\sqrt{v_{it}}\, dZ_{it} \tag{1}$$

with:

$$dv_{it} = \kappa_i \left(\theta_i - v_{it}\right) dt + \sigma_i \sqrt{v_{it}}\, dW_{it}$$

where θ_i represents the long-term mean corresponding to the instantaneous variance factor i (for $i=1, 2$), κ_i denotes the speed of mean reversion and, finally, σ_i represents the volatility of the variance factor i. For analytical convenience, let us rewrite the previous equation as follows:

$$dv_{it} = (a_i - b_i v_{it})\, dt + \sigma_i \sqrt{v_{it}}\, dW_{it} \tag{2}$$

where $b_i = \kappa_i$ and $a_i = \kappa_i\theta_i$. In Eqs. (1) and (2) Z_{it} and W_{it} are Wiener processes such that:

$$dZ_{it} dW_{jt} = \begin{cases} \rho_i dt \text{ for } i = j \\ 0 \text{ for } i \neq j \end{cases}$$

On the other hand, Z_{1t} and Z_{2t} are uncorrelated. In addition, W_{1t} and W_{2t} are also uncorrelated. The single-factor Heston (1993) model can be obtained as a particular case of the two-factor specification considering only one volatility factor. The multifactor specification of Eqs. (1) and (2) accounts for a richer variance–covariance structure. In particular, the conditional variance of the return process is:

$$V_t := \frac{1}{dt} Var(dY_t) = \sum_{i=1}^{2} v_{it}$$

whereas, as shown by Christoffersen et al. (2009), the correlation between the asset return and the variance process is:

$$\rho_{A_t V_t} := Corr\left(dY_t, dV_t\right) = \frac{\rho_1\sigma_1 v_{1t} + \rho_2\sigma_2 v_{2t}}{\sqrt{\sigma_1^2 v_{1t} + \sigma_2^2 v_{2t}}\sqrt{v_{1t} + v_{2t}}}$$

Importantly, two-factor specification, unlike the single-factor stochastic volatility models, allows for stochastic correlation between the asset return and the variance process. Another advantage of the two-factor model with respect to single-factor specifications is that it provides more flexibility to model the volatility term structure. In this sense, the two-factor model is able to generate more flexible patterns corresponding to the term structure of credit spreads.

2.1. Pricing credit spreads and default probabilities

In this section I follow the methodology of Lewis (2000) and da Fonseca et al. (2007) to calculate option prices efficiently in terms of the generalized Fourier transform associated with the payoff function and with the asset return. In this sense, let us consider a generic payoff on the terminal value of the underlying asset, at time $t=T$, under the risk-neutral probability measure $w(Y_T)$. From the Fundamental Theorem of Asset Pricing we have that the time $t=0$ price of this option, denoted OP_0, is given by:

$$OP_0 = e^{-rT} E_\Theta\left[w(Y_T)\right] = e^{-rT}\int_{\mathbb{R}} w(Y_T)\,\delta_T(Y_T)\, dY_T \tag{3}$$

where $\delta_T(Y_T)$ is the risk-neutral density function of Y_T. The Laplace transform of the asset return is defined as:

$$\Psi(\lambda; Y_0, T) := E_\Theta\left[e^{\lambda Y_T}\right] = \int_{\mathbb{R}} e^{\lambda Y_T}\delta_T(Y_T)\, dY_T \quad \lambda \in \mathbb{R}$$

On the other hand, the Fourier transform corresponding to $w(Y_T)$ is given by:

$$\widehat{w}(z) = \int_{\mathbb{R}} e^{izY_T} w(Y_T)\, dY_T \quad z \in \mathbb{C}$$

with

$$w(Y_T) = \frac{1}{2\pi}\int_{\chi} e^{-izY_T}\,\widehat{w}(z)\, dz$$

where $\chi \subset \mathbb{C}$ is the admissible integration domain in the complex plain corresponding to the generalized Fourier transform associated with the payoff function $\widehat{w}(z)$ and where $i^2 = -1$. Substituting previous expression in Eq. (3) yields:

$$OP_0 = \frac{e^{-rT}}{2\pi}\int_{\chi} \Psi(\lambda = -iz; Y_0, T)\,\widehat{w}(z)\, dz \tag{4}$$

where we have used the Fubini theorem. From the previous equation, to obtain a semi-closed-form solution for the option price we have to calculate the Laplace transform of the asset return, as well as the Fourier transform associated with the payoff function.

2.1.1. The Laplace transform of the asset return

Marabel Romo (2013) shows that, under the risk-neutral measure Θ, the Laplace transform associated with the two-factor specification $\Psi(\lambda\,; Y_0, T)$ is given by:

$$\Psi\left(\lambda; Y_0, T\right) = e^{B(\lambda,T) + \sum_{i=1}^{2} M_i(\lambda,T) v_{i0} + \lambda Y_0} \tag{5}$$

where $M_i(\lambda, T)$ for $i=1, 2$, is given by:

$$M_i(\lambda, T) = \frac{b_i - \lambda\rho_i\sigma_i + \alpha_i}{\sigma_i^2}\left[\frac{1 - e^{T\alpha_i}}{1 - \beta_i e^{T\alpha_i}}\right]$$