The Spanish Review of Financial Economics 10 (2012) 41–52

The Spanish Review of Financial Economics

www.elsevier.es/srfe

Article

Multiplicity in financial equilibrium with portfolio constrains under the generalized logarithmic utility model

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ARTICLE INFO

Article history:
Received 6 March 2012
Accepted 3 April 2012
Available online 19 July 2012

JEL classification:
G12
D52

Keywords:
General financial equilibrium
Portfolio constraints
Generalized logarithmic utility
Multiplicity of equilibria

ABSTRACT

Previous research on the effects of constraints to take unbounded positions in risky financial assets shows that, under the logarithmic utility function, multiplicity of equilibrium may emerge. This paper shows that this result is robust to either constant, decreasing or increasing relative risk aversion obtained under the generalized logarithmic utility function.

1. Introduction

The effect of portfolio constraints and capital market imperfections on financial asset pricing equilibrium is a key topic on financial economics. This type of analysis is especially relevant after the turmoil experienced by financial markets in the last three years.

There are three main strands of literature dealing with the effects of portfolio constraints on financial equilibrium. A recent particularly relevant research analyzes the impact of portfolio constraints on the international propagation of adverse shocks. Pavlova and Rogobon (2008) show how portfolio constraints amplify stock price fluctuations among international stock exchange markets. As expected, absent of portfolio constraints, all co-movements are due to the common stochastic discount factor and the term of trade channels. However, once portfolio constraints are in place, the authors characterize a dynamic equilibrium model in which an excess co-movement in stock prices relative to the unconstrained economy naturally arises. Moreover, they are able to associate their results with the contagion phenomenon previously studied in literature.

The second strand is the literature on asset pricing models with different types of frictions and market imperfections. The effects of portfolio constraints on equilibrium asset and consumption allocations typically include short-sales, borrowing, liquidity constraints, and restricted participation. Examples are Jarrow (1980), He and Pearson (1991), He and Modest (1996), Heaton and Lucas (1996), Detemple and Murphy (1997), Basak and Cuoco (1998), Basak and Croitoru (2000), Kogan and Uppal (2001), Detemple and Serrat (2003), Scheinkman and Xiong (2003), Gallmeyer and Hollifield (2008), and Bhamra and Uppal (2009). An overall conclusion of this literature seems to indicate that short-sale constraints may lead to higher equity volatilities whereas borrowing-constrained equilibria typically leads to lower equity volatilities,[1] and that frictions generate a wedge between the stochastic discount factor and asset prices large enough to make some well-known empirical puzzles compatible with equilibrium in financial markets.

Finally, a third related strand, which is especially relevant for this paper, investigates the effects of portfolio constraints on the multiplicity of financial equilibrium. Basak et al. (2008) (BCLP hereafter) are the first to investigate the extent to which portfolio constraints to take unbounded positions in risky assets generate multiplicity or even indeterminacy of equilibria. They show that the introduction of this type

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[1] As an example of the importance of this topic, a recent empirical analysis of the effects of short-sales restrictions imposed during the 2007–2009 financial crises due to Beber and Pagano (2010) show that short-sales regimes were detrimental for liquidity, slow down price discovery, and failed to support stock prices.

2173-1268/$ – see front matter
http://dx.doi.org/10.1016/j.srfe.2012.04.002

of constraints increase the number of equilibria in the economy. In particular, they demonstrate that under potentially complete markets and portfolio constraints, there may be a finite number of additional equilibria over and above the efficient original financial equilibrium. Moreover, under incomplete markets and portfolio constraints, there is also a continuum of them, with consumption allocations varying across this continuum. Indeed, as BCLP (2008) point out, under these circumstances, there may be robust real indeterminacy of equilibrium.

The finding that portfolio constraints may expand the set of equilibria, even though by itself it does not imply equilibrium indeterminacy, is a particularly relevant result because it may help to understand, within a rational framework, the large fluctuations of asset returns which are difficult to explain simply by changes in economic fundamentals.[2] If a financial market with binding portfolio constraints admits more than one but still a finite number of equilibria for the same economic fundamentals, variability of stock prices could be entirely due to investors' expectations. Thus, excess volatility or market crashes may be explained by rational expectation-generated phenomena, with investors selecting a particular equilibrium over another.

BCLP consider a simple pure-exchange two-period model, with two goods, two households, two states of nature (in the complete markets case) and two stocks paying off in units of the goods. In the incomplete markets case an additional state is incorporated, leaving the number of financial assets and the number of goods unchanged.

Interestingly, households are characterized as heterogeneous agents in the sense of having different marginal propensity to consumption, and different initial endowments specified in terms of shares of stocks, not goods. More importantly, only one of the household faces a portfolio constraint to take unbounded positions on the holdings of one of the risky assets. Otherwise, no more restrictions are considered; in particular, short sales are permitted. Unfortunately, however, the household preferences are described by a Cobb–Douglas log-linear utility function. Indeed, as BCLP recognize, this may be a rather restrictive assumption since this utility functions presents decreasing absolute risk aversion, but constant relative risk aversion. While the issue of the realistic sign of absolute risk aversion has been settled for a long time, the direction of relative risk aversion remains an open question. This suggests that a satisfactory model should be tolerant of different attitudes of relative risk aversion.

The contribution of this paper is to investigate the robustness of equilibrium multiplicity with portfolio constraints under a more general and flexible utility function.[3] Along these lines, Rubinstein (1975) convincible argues that a successful asset pricing model should require decreasing absolute risk aversion, and tolerate increasing, constant, or decreasing relative risk aversion. He shows that the generalized logarithmic utility function is a particularly attractive model since it satisfies this requirement and, at the same time, it allows a pricing expression for an uncertain intertemporal cash flow stream even when this is serially correlated over time. Furthermore, the asset pricing model under this type of preferences assumes no exogenous intertemporal stochastic process of asset prices. For a given household h, and a single consumption good, the generalized logarithmic utility function is given by,

$$u_h(C_h) = \log(C_h + J_h), \tag{1}$$

where $u_h(\cdot)$ is the utility of household h, C_h is the consumption of the available good, and J_h is an exogenous taste parameter that may take different signs. This is the key parameter that captures heterogeneity among households since it simultaneously tolerates increasing, constant or decreasing relative risk aversion depending upon J_h is positive, zero or negative, respectively.[4] Therefore, J_h will be referred to as the measure of household risk-preference; the higher J_h, the more risk preferring the household. It is also the case that, when $J_h \leq 0$, the household will never consume for $C_h \leq -J_h$, since such low levels of consumption have infinite disutility. Hence, when $J_h \leq 0$, $-J_h$ may be interpreted as the subsistence level of consumption. Finally, this utility function belongs to the Hyperbolic Absolute Risk Aversion (HARA) or linear risk tolerance class of tastes and it represents the solution to the differential equation,

$$-\frac{u'(c_h)}{u''(c_h)} = J_h + Bc_h, \tag{2}$$

for $B = 1$.[5]

In this paper, we extend the model of BCLP (2008) when the households are characterized by the log-linear generalized logarithmic utility function with different propensities to consume. For the case of two households, and two goods, the utility function is given by,

$$u_h(C_h^1, C_h^2) = \alpha_h^1 \log(C_h^1 + J_h^1) + \alpha_h^2 \log(C_h^2 + J_h^2), \tag{3}$$

where, as before, u_h is the utility of each household h characterized by decreasing absolute risk aversion; C_h^g is the consumption of the good g by household h; α_h^g is the marginal propensity to consume the good g by household h, and J_h^g is the taste parameter representing either increasing ($J_h^g > 0$), constant ($J_h^g = 0$), or decreasing ($J_h^g < 0$) relative risk aversion.

The main result of this paper is that the multiplicity of equilibria remains for all J_h^g. This implies that large movements in financial markets may be related to the effects of portfolio constraints on equilibrium prices.

The rest of the paper is organized as follows. Section 2 describes the basic model and shows the results in the complete markets case. Section 3 analyzes the effects of incomplete markets. Finally, Section 4 concludes the paper.

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[2] For example, Cutler et al. (1989) argue in a very influential paper that most of the large market moves after the Second World War cannot be apparently explained by releases of economic or fundamental information. On the other hand, Cochrane (2011) argues that strong time-varying expected returns seem to be able to justify most of the previously unexplained price fluctuations.

[3] Specifically, BCLP in their seminal paper argue that "though we believe that, for the most part, our central results are robust to local perturbations of utility functions around the specific log-linear functions we employ here. But this conjecture remains to be fully, seriously investigated."

[4] Of course, when J_h equals zero, we have the typical log utility function with decreasing absolute risk aversion, and constant relative risk aversion, employed by BCLP (2008).

[5] Note that for $B \neq 0, 1$ we obtain the generalized power utility function, while for $B = 0$, we have the negative exponential utility function.