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Approximate value-at-risk calculation for heterogeneous loan portfolios: Possible enhancements of the Basel II methodology☆

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ABSTRACT

This paper presents three possible methods by which the credit value at risk estimates coming from the Basel II IRB approach can be significantly improved upon. The feasibility of the suggested approaches is substantiated by applying it to an exemplary model portfolio.

1. Introduction

For many banks, the possible failure of borrowers to make their interest and principal payments is the most important source of financial risk. Individual banks, in turn, are closely linked to each other, as well as to non-bank institutions, through a dense network of loans, deposits, and (in some cases) equity shareholdings. The losses induced by the failure of a single institution are therefore not confined to its respective shareholders but will partially spill over to others. These "external" costs of insolvency are usually not included in the risk/return considerations of individual decision makers. Hence, fears have

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been mounting that the risk limitation measures voluntarily undertaken by market participants might not suffice to rule out the possibility that, in extreme cases, an initial failure of few large borrowers might trigger an avalanching succession of further defaults and a severe macroeconomic downturn: According to Shireff (2005), the Bank of England estimates the average cost of a full-blown banking crisis for the country concerned at 16% of GDP.

In response to this challenge, supervisory authorities and central banks from several countries have joined efforts with the Bank for International Settlements to form the Basel Committee on Banking Supervision. Its task is to develop and implement rules for the measurement and limitation of risks incurred by banks, as well as minimum standards for their respective capital endowment. A milestone in this ongoing process is the New Basel Capital Accord (Basel II; see Basel Committee, 2003). As to credit risk, the Accord provides a mathematical formula by which data on internal ratings, exposure amounts, loan maturities, and debtor size classes are used to compute value at risk (VaR) estimates from which the portfolio-specific capital requirements are derived. This model has been termed the "Internal Ratings-Based" (IRB) approach in order to separate this formula from simpler, yet less informative, alternatives.

A problem with the Basel II model is that in order to remain analytically tractable, it has to assume the existence of an infinitely fine-grained loan portfolio. This premise is clearly unrealistic for most real-world portfolios consisting, at least in part, of relatively large loans to companies or governments. As a consequence, the Basel II model often tends to produce biased estimates of the true credit VaR. This paper describes three alternative computational techniques by which the extent of this unfavourable distortion of results can be diminished considerably without sacrificing computational tractability. To this end, we proceed by first recapitulating the theoretical basis of the credit portfolio model of the Basel II approach (Section 2.1), and characterise the loss distribution it implies under the assumption of perfect granularity (Section 2.2). In Section 2.3, we assess the accuracy of this implied loss distribution by applying the Basel II model to a hypothetical test portfolio, and compare the outcome with the "true" loss distribution computed via Monte Carlo simulation. The three alternative calculation methods we suggest, and the results of their application to the test portfolio, are detailed in Section 3. Section 4 concludes the paper.

2. Problem formulation

2.1. Theoretical foundation: the Merton/Vasicek approach

The theoretical foundation of the Basel II is Vasicek's (2002) adaptation of Merton's (1974) approach to the pricing of corporate debt. In this framework, a company is assumed to default if the market value of its assets drops below the face value of its liabilities. To capture this idea formally, the model specifies a normalised asset return y_i for each debtor i, which depends on a macroeconomic performance index z common to all obligors, and on a debtor-specific scalar ε_i:

$$y_i = w_i \cdot z + \sqrt{1 - w_i^2} \cdot \varepsilon_i \text{ with } \varepsilon_i, z \sim N(0, 1) \text{ and } E(\varepsilon_i \cdot z) = 0 \text{ for all } i = 1, \ldots, N. \quad (1)$$

The parameter w_i measures the extent to which the credit quality of debtor i is influenced by the common macroeconomic factor z. It is supposed that its particular values for debtors 1 to N are known, which is a strong but defendable assumption because these parameters can be calibrated on the grounds of historical default frequency data (see, e.g., Puzanova and Siddiqui, 2005).

An obligor defaults if y_i drops to or below a given, debtor-specific threshold τ_i. Let D_i denote an indicator which becomes 1 if the debtor defaults, and zero otherwise. Then, the relationship between D_i, z, and ε_i can be expressed as

$$D_i = I(w_i \cdot z + \sqrt{1 - w_i^2} \cdot \varepsilon_i \leq \tau_i). \quad (2)$$

The probability of default for debtor i, denoted by p_i, is assumed to be known, so that the default threshold τ_i equals

$$\tau_i = \Phi^{-1}(p_i). \quad (3)$$