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Pricing and hedging of arithmetic Asian options via the Edgeworth series expansion approach

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Abstract

In this paper, we derive a pricing formula for arithmetic Asian options by using the Edgeworth series expansion. Our pricing formula consists of a Black-Scholes-Merton type formula and a finite sum with the estimation of the remainder term. Moreover, we present explicitly a method to compute each term in our pricing formula. The hedging formulas (greek letters) for the arithmetic Asian options are obtained as well. Our formulas for the long lasting question on pricing and hedging arithmetic Asian options are easy to implement with enough accuracy. Our numerical illustration shows that the arithmetic Asian options worths less than the European options under the standard Black-Scholes assumptions, verifies theoretically that the volatility of the arithmetic average is less than the one of the underlying assets, and also discovers an interesting phenomena that the arithmetic Asian option for large fixed strikes such as stocks has higher volatility (elasticity) than the plain European option. However, the elasticity of the arithmetic Asian options for small fixed strikes as trading in currencies and commodity products is much less than the elasticity of the plain European option. These findings are consistent with the ones from the hedgings with respect to the time to expiration, the strike, the present underlying asset price, the interest rate and the volatility.

JEL classification: G10; G12; G13

Keywords: Arithmetic Asian option; Edgworth series expansion; Cumulant; Elasticity; Hedge

1. Introduction

Banker Trust's Tokyo office first issued the arithmetic Asian option for pricing average options on crude oil contracts in1987.[e] The payoff of arithmetic Asian options depends on the arithmetic average price of the underlying

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[e] Boyle and Emanuel[11] first introduce the Asian options. Ingersoll[22] gives the first formulation of a PDE with the T-forward measure. Rogers and Shi[23] formulated this problem using a stock as a numeraire.

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asset, commonly traded as currencies and commodity products. The averaging procedure reduces the significance of the closing price and reduces the effects of abnormal price changes at the maturity of the option. Turnbull and Wakeman[1] point out that the arithmetic Asian options provide a way to ameliorate any possible price distortions that might arise because of a lack of depth in the market of the underlying asset.

Pricing arithmetic Asian options is a difficult issue in finance for several decades. In general, there are two different approaches to find the option pricing formula by (1) solving the corresponding partial differential equation with boundary conditions, and (2) evaluating the option by the known distribution for the underlying price. See Boyle and Boyle[2] for the history and evolution of Asian options. There are different numerical approaches to price the arithmetic Asian options, Linetsky[3] provides a pesudo closed form solution with numerical finding on certain eigenvalues, Hoogland and Neumann[4] and Vecer[5] price the Asian options with simple integral representations of Laplace transformations and with the use of Whittaker functions, see also Alziary, Décamps and Koehl,[6] Chen and Lyuu,[7] Elshegmani, Ahmed and Jaaman[8] and Elshegmani, Ahmed and Jaaman, Zakaria (2011), Sun et al[9] and references therein.[f]

Shreve[10] shows that there is a single PDE with boundary conditions to price the Asian call option (Theorem 7.5.1 and Theorem 7.5.3 of Shreve[10] provide PDEs for the Asian options.). There is no analytic solution for the partial differential equation (PDE) of the arithmetic Asian option. Solving the PDE of the arithmetic Asian option has been an equivalently outstanding issue in mathematical finance for a long time since the PDE is a degenerate partial differential equation in three dimension, and the numerical solution of this PDE is not very accurate due to the low volatility level of the arithmetic Asian option. Elshegmani, Ahmed and Jaaman, Zakaria (2011) transform the PDE of arithmetic Asian option to a parabolic equation with constant coefficients and obtain the analytic solution of the arithmetic Asian option PDE. Their closed form analytical solution for the arithmetic Asian option PDE is lack of tractability (in terms of solutions for a heat equation *without initial condition*), and does not satisfy the necessary boundary conditions for the Asian option. Their solution is only for the PDE of the arithmetic Asian option, *not for the boundary conditions required from the arithmetic Asian options*. The solution form in their Formula (2.2) of Elshegmani, Ahmed and Jaaman, Zakaria (2011) does not satisfy any boundary condition (2.5)−(2.7) for the arithmetic Asian option. Vecer[5] derives the Black-Scholes representation of the value of the Asian option in terms of the corresponding probabilities that the option will end up in the money, and both the price and hedge are computed numerically from the partial differential equations, where the PDEs are in their simplest form under the respective numeraire measures.

A closed-form formula for pricing and hedging of arithmetic Asian options does not exist since this exotic Asian option is introduced by Boyle and Emanuel.[11] This is due to the difficult fact that the average of lognormal random variables is no longer log-normally distributed.[g] The arithmetic Asian options are path-dependent options over the averaging period. This makes binomial tree approach infeasible. Various numerical methods to price arithmetic Asian options have been studied extensively in the literature to compete with the efficiency and accuracy. Kemna and Vorst[12] show that the combined process of the underlying asset and its average is not Gaussian in character which implies that it is impossible to obtain an explicit formula for the arithmetic Asian option pricing.[h] Kemna and Vorst[12] obtain an explicit pricing formula for geometric Asian options and apply the Monte Carlo simulation with variance reduction to price geometric Asian options. Geman and Yor[13] develop some efficient analytic tools to get quasi-explicit pricing formulas in terms of Laplace transformations, and evaluate the Laplace transform of an arithmetic Asian option by a number of standard methods to invert numerically. Milevsky and Posner[14] use moment-matching methods to fit the distribution function of the arithmetic average of the underlying asset with a reciprocal gamma distribution function. Ju[15] makes use of the Taylor expansion of the ratio of the characteristic function of the average to that of the approximating lognormal variable near zero volatility. Dufresne[16] utilizes Laguerre series expansions to regulate the density of the integral of a geometric Brownian motion and uses the results to get another expression for the price of an arithmetic Asian option.

Pricing and hedging of arithmetic Asian options in a tractable manner is not only useful in the financial practice, but also beneficent in mathematical finance. The objective of this paper is to provide a reasonable answer to this challenge

[f] Vecer[24] does have the standard approach used in practice to cover discrete monitoring. Many formulas in literature are less efficient with various approximations or Monte Carlo simulations.

[g] A random variable is lognormal if its logarithm is normally distributed. Standard option pricing methods that rely on the lognormal assumption, cannot be applied in the arithmetic Asian option.

[h] This no-Gaussian process indicates the conditional expectation with respect to the underlying asset price, its arithmetic average and the present time cannot be evaluated explicitly. Arnold (1974) in his book *Stochastic differential equations, theory and applications* gives an account of discussion on this point.