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Frontiers in financial dynamics

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ABSTRACT

In recent decades, mathematical motivated financial models have been used to understand the complexity and intermittent nature of financial market instruments. Typically, applied mathematics models a physical system by specifying and quantifying the physical laws to which the process should theoretically conform. Such theoretical models are often represented as differential equations. The solutions of these differential equations have been shown to have poor compliance with observed financial data which has been attributed to difficulties in correctly estimating the parameters of the differential equation. Generalised smoothing provides a comprehensive evaluation of financial dynamics as it accurately estimates data driven parameters for differential equations and produces a fitted curve that incorporates the theoretical specifications implied by the differential equation while adhering to the observed financial data. This article demonstrates the merit for a generalised smoothing approach to modeling financial dynamics by examining instantaneous forward yield curves within a generalised smoothing framework.

1. Introduction

In recent decades, mathematical motivated financial models have been used to understand the complexity and intermittent nature of financial market instruments. The ad hoc use of these models has caused increasing concern within the financial sector and has been the cause of much speculation for the merit of these models in finance. Typically applied mathematics models a physical system by

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specifying and quantifying the physical laws to which the process should theoretically conform. Such theoretical models are often represented as differential equations. However, differential equations have been shown to have poor compliance with observed financial data for systems that cannot be as well controlled as the model suggests (Filipovic, 1999). This has been attributed to difficulties in correctly parameterising systems establishing the evolution of the financial instrument over a pre-specified domain (Dia and Singleton, 2003).

Generalised smoothing draws on well established statistical and applied mathematical concepts, to obtain a procedure that efficiently produces optimal estimates of the parameters of the specified differential equation. Generalised smoothing has been the subject of many publications, most notably: Heckman and Ramsay (2000), Ramsay and Silverman (2005), Ramsay et al. (2007), Cao and Ramsay (2007). Generalised smoothing falls in the realm of functional data analysis (FDA). FDA treats the data ($\mathbf{y}$, $\mathbf{x}$), where $\mathbf{y}$ is a vector of observational variables and $\mathbf{x}$ is a vector of the corresponding locations, as originating from an underlying functional entity $f(x)$. The almost universal approach is to approximate $f(x)$ by a basis function expansion $f(x) \approx \mathrm{B}(x)\mathbf{c}$, where $\mathrm{B}(x)$ are the basis functions and $\mathbf{c}$ are the corresponding coefficients. Typically, it is sufficient to only estimate the continuous function $f(x)$ over a set of plausible predictor values $\mathbf{x}$, thus, $f(\mathbf{x}) = \mathbf{B}(\mathbf{x})\mathbf{c}$. Prior knowledge of the form of the functional entity $f(x)$ is incorporated into the modeling procedure in the form of a model based penalty. The model based penalty is represented by a differential operator $Lf(x)$ which is an ODE whose solution is the theorised form of $f(x)$.

Generalised smoothing is concerned with the minimisation of the following penalised residual sum of squares:

$$\begin{aligned}\mathrm{PENSSE}(\mathbf{y}|\mathbf{x}, \mathbf{c}, \boldsymbol{\beta}, \lambda) &= \|\mathbf{y} - f(\mathbf{x})\|^2 + \lambda \times \int \|Lf(x)\|^2 \mathrm{d}x, \\ &= \|\mathbf{y} - \mathbf{B}(\mathbf{x})\mathbf{c}\|^2 + \lambda \times \mathbf{c}'\underbrace{[\int L\mathrm{B}(x)L\mathrm{B}(x)' \mathrm{d}x]}_{\mathbf{R}}\mathbf{c}.\end{aligned} \tag{1}$$

This minimisation involves the estimation of three types of parameters: local parameters $\mathbf{c}$ which are the coefficients of the basis function expansion $\mathbf{B}(x)\mathbf{c}$ defining the smooth $f(x)$, global parameters $\boldsymbol{\beta}$ which are the parameters of the differential equation defining the linear differential operator $Lf(x)$ and the complexity parameter λ which controls the fitted curves adherence to the linear differential operator $Lf(x)$. Carey et al. (2012) have shown that for linear ordinary differential equations (ODE's) the model based penalty $\mathbf{R}$ can be written explicitly in terms of the unknown global parameters $\boldsymbol{\beta}$. This approach has several compelling advantages: the minimisation procedure devolved in Carey et al. (2012) is more efficient than preexisiting methods and the computation of the standard errors of the local and global parameters is parsimonious.

In this article, we demonstrate the merit for a generalised smoothing approach for estimating the parameters of ODEs. In particular this article examines a generalised smoothing approach for modeling yield curves.

Nelson and Siegel (1987) concede that typical yield curve shapes identified in Wood (1983) and Malkiel (1966) are generated by the solution to the second order linear homogenous differential equation with constant coefficients.

$$\frac{d^2y}{dx^2} + \beta_1\frac{dy}{dx} + \beta_0 y = 0.$$

The parameters of this differential equation (β_0, β_1) quantify the level and slope of the yield curve which are perceived as indicators of monetary policy. The level of the yield curve indicates long-run inflation expectations while the slope of the yield curve provides information on the current state of monetary policy (growth/recession). Similarly, the parameters of the solution to the differential equation quantify the strengths of the short term, medium term and long term components of the yield curve. The accurate estimation of these parameters is crucial in financial and economic analysis of fixed income.