Research in International Business and Finance 23 (2009) 107–116

Contents lists available at ScienceDirect

Research in International Business and Finance

journal homepage: www.elsevier.com/locate/ribaf

Portfolio optimization with CVaR under VG process☆

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ARTICLE INFO

Article history:
Received 11 October 2007
Received in revised form 18 July 2008
Accepted 29 July 2008
Available online 5 August 2008

Keywords:
Portfolio
CVaR
Variance Gamma
Copula
Monte Carlo

ABSTRACT

Formal portfolio optimization methodologies describe the dynamics of financial instruments price with Gaussian Copula (GC). Without considering the skewness and kurtosis of assets return rate, optimization with GC underestimate the optimal CVaR of portfolio. In the present paper, we develop the approach for portfolio optimization by introducing Lévy processes. It focuses on describing the dynamics of assets' log price with Variance Gamma copula (VGC) rather than GC. A case study for three Indexes of Chinese Stock Market is performed. On application purpose, we calculate the best hedge positions of Shanghai Index (SHI), Shenzhen Index (SZI) and Small Cap Index (SCI) with the performance function CVaR under VG model. It can be combined with Monte Carlo Simulation and nonlinear programming techniques. This framework is suitable for any investment companies.

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1. Introduction

Since Markowitz published his seminal work which introduces mean/variance risk management framework in 1952, there has been lots of theoretical and empirical work on portfolio optimization with different utility functions, risk measures and constraints.

Merton (1969, 1971) pioneered in applying continuous-time stochastic models to the study of financial markets (without transaction costs). He showed that the optimal investment policy of a constant relative risk aversion (CRRA) investor is to keep a constant fraction of total wealth in the risky asset during the whole investment period. The introduction of proportional transaction costs to Merton's model was first accomplished by Magill and Constantinides (1976), Davis and Norman (1990) stud-

☆ Project (No. Y604137) supported by ZheJiang Natural Science Foundation.

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doi:10.1016/j.ribaf.2008.07.004

ied the problem with transaction costs for an infinite time. A further work carried out by Shreve and Soner (1994) fully characterizes the optimal polices relying on the concepts of viscosity solution to Hamilton–Jacobi–Bellman (HJB) equations. Liu and Loewenstein (2002) considered an optimal problem with a stochastic time horizon following Erlang distribution. Some other papers about this are Dai and Yi (2006), Sun et al. (2007).

Measures of risk have a crucial role in optimization under uncertainty, especially in coping with the losses that might be incurred in finance of the insurance industry. Value at Risk, or VaR for short, is one of the most popular measures due to its simplicity, which has achieved the high status of being written into industry regulations. But this risk measure is not always sub-additive, nor convex. Artzner et al. (1999) proposed the main properties that a risk measures must satisfy, thus establishing the notion of coherent risk measure.

Conditional Value-at-Risk, or CVaR for short, is defined as the weighted average of VaR and losses strictly exceeding VaR for general distributions, see Rockafellar and Uryasev (2002). The CVaR risk measure has been proved to be a coherent risk measure in Pflug (2000); see also Rockafellar and Uryasev (2001), Acerbi et al. (2001), Acerbi and Tasche (2002). After that, other classes of measures have been proposed, each with distinctive properties: Conditional Drawdown-at-risk (CDaR) in Chekhlov et al. (2000), ES in Acerbi et al. (2001), convex measures in Follmer and Shied (2002), spectral measures in Acerbi and Simonetti (2002), and deviation measures in Rockafellar et al. (2006).

A simple description of the approach for minimizing CVaR and optimization problems with CVaR constraints can be found in Chekhlov et al. (2000). Gaivoronski Pflug (2000) have found that in some cases optimization of VaR and CVaR may lead to quite different portfolios. Rockafellar and Uryasev (2000) demonstrated that linear programming techniques can be used for optimization of the Conditional Value-at-Risk (CVaR) risk measure. Several case studies showed that risk optimization with the CVaR performance function and constraints can be done for large portfolios and a large number of scenarios with relatively small computational resources. A case study on the hedging of a portfolio of options using the CVaR minimization technique is included in Rockafellar and Uryasev (2000). Also, the CVaR minimization approach was applied to credit risk management of a portfolio of bonds, see Andersson et al. (1999). This paper extends the CVaR minimization approach in Rockafellar and Uryasev (2000) to other classes of problems with CVaR functions. Further moer, CVaR minimization approach was extended to derivative portfolio hedging, see Alexander et al. (2003), and with transaction cost in Alexander et al. (2006).

In those papers, they focus on describing the dynamics of assets log price with multiple Weiner process which is continuous and normal distribution. Unfortunately, as documented in a considerable number of papers written by academics and practitioners, both normality and continuity assumptions are contradicted by the data in several pieces of evidence. As noted by Fama (1965), return distributions of financial instruments are more leptokurtic than normal distributions and tend to be exhibit “fat tails”. This phenomenon becomes particularly clear on high frequency data and be more accentuated when the holding period becomes shorter. In these aspects the VG process, which was first introduced in financial modeling by Madan and Seneta (1990) to cope with shortcomings of Black–Scholes model, is superior to the Weiner process.

By introducing extra parameters, Variance Gamma (VG) process has a number of good mathematical properties and has been proven to explain a number of economic findings. Mathematically, the distributions have nice properties such as leptokurtic and fat tails. Economically, Madan et al. (1998) shows that their model is able to explain the well documented biases “volatility smile” in equity options. Moreover, Cariboni and Schoutens (2004) shows that their VG model for CDOs pricing fits to a variety of single name credit curves.

In the present paper, we drop the limitations of normality and continuity assumptions and complete the foundations for our methodology. We extend the portfolio optimization framework by describing the dynamics of assets' log price with VG copula rather than Gaussian copula. We find that CVaR based on the classical Multiple Normal Distribution underestimate the risk of financial instruments. As a result, We suggest considering skewness and kurtosis into portfolio optimization framework by introducing Variance Gamma copula to describe instrument dynamics.

The outline of this paper is as follows. The Variance Gamma process is summarized in Section 2 and its properties are presented and discussed in detail. In Section 3, we reformulate the frameworks of

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