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Comparative study of gravity-driven discharge from reservoirs with translationally invariant and fractal pore networks

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ABSTRACT

We perform a comparative study of water discharge from Euclidean and fractal reservoirs. The Euclidean reservoir is modeled by a filter with a translationally invariant cubic network of straight square channels. As a model of fractal reservoir with the scale invariant pore network we use a pre-fractal inverse Menger sponge. The gravity-driven discharge experiments were carried out with two different setups. In the first setup, a porous filter is fixed in container above a lateral exit duct. In the second setup, the exit duct is connected to one of lateral exits of the pore network. In both cases, we found that the squared outflow velocity is a linear function of water table elevation above exit orifice. Hence, the water discharge is governed by the Bernoulli's principle rather than by the Darcy's law. Furthermore, we found that filters placed above the exit duct have no effect on the discharge rate, because absolute permeabilities of both filters are large enough to supply water into the volume between filter's bottom and exit orifice. Conversely, when the exit duct is connected to the lateral exit of the pore network, the water discharge from the inverse Menger sponge is faster than from the Euclidean reservoir. Experimental data fittings with a modified Torricelli's equation suggest that a loss of mechanical energy of flowing water in the fractal pore network is less than in the translationally invariant network of channels. This finding reveals that an effective wetted area along a flow path of least resistance in the self-similar pore network is smaller less than in the periodic network of straight channels. So, the fractal features of the pore network can have a strong impact on hydrological processes in natural reservoirs and karst aquifers which commonly possess statistical scale invariance.

1. Introduction

A gravity-driven discharge from reservoirs is one of the long-standing problems in the fluid mechanics and hydrology (Brown, 2002; Clanet, 2000). Classical examples include water discharges from a hollow container through a short duct (Joye and Barrett, 2003; Libii, 2003) and from karst aquifers through springs (Chang et al., 2015; Fiorillo and Guadagno, 2012). In this background, there were identified different regimes of the gravity-driven discharge from porous reservoirs, which are distinguished by different types of dependence of volumetric discharge rate Q on hydraulic head h. Specifically, the Darcy's regime is characterized by linear behavior $Q \propto kh$, where k is the absolute permeability of porous medium (Brown, 2002), whereas in the Torricelli's regime of the gravity-driven discharge from the porous reservoir $Q^2 \propto h$ (Maramathas and Boudouvis, 2009).

The absolute permeability of medium is determined by a network of conducts (pores, cracks, etc.) embedded in a low permeable matrix. Historically, the pore and fracture networks were first modeled using translationally invariant networks (see, for review, Raoof and Hassanizadeh, 2010; Stevanovic, 2015). Since then, it was recognized that pore and fracture networks in natural reservoirs and karst aquifers are commonly scale invariant, rather than translationally invariant (Fac-Beneda, 2013; Maramathas and Boudouvis, 2006; Veltri et al., 1996). This has stimulated the use of fractal concepts to model hydrographic systems (Berkowitz, 2002; Castaing et al., 2002). Although real pore networks are statistically self-similar only over a bounded range of length scales, the fractal geometry allows to quantify network features by a set of fractional dimensionalities (Balankin, 2015, 2018; Gouyet, 1996). Specifically, the scale invariance of the pore network implies that the volume of pore space increases with the sample size L as $V_P = (L/l_0)^D$, where D is the fractal (e.g. self-similarity or Hausdorff) dimension of the network, while l_0 is the minimum pore size (Feder,

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1988). Consequently, if $D < 3$, the medium porosity $\varphi = V_P/L^3$ is scale dependent $\varphi = (L/l_0)^{D-3}$ (Fallico et al., 2010). The absolute permeability k is a purely geometric property (Becker, 2018; Dullien, 1992). Accordingly, the Darcy's flow rate is stoutly dependent on the fractal properties of pore network (see Balankin and Espinoza-Elizarraraz, 2012; Chang and Yortsos, 1990; and references therein).

Conversely, the Torricelli's theorem states that the outflow velocity u is independent of the reservoir geometry (Bueche, 1972; Mitchell, 1945). On the other hand, it was established that fluid wetting in the exit duct leads to decrease the outflow velocity from the hollow container (Ferrand et al., 2016; Luque-Escamilla, 2009). Therefore, one may expect that the wetting phenomena in the pore network may also affect the Torricelli's discharge rate

$$Q = C_c s_O u \tag{1}$$

where s_O is the cross-sectional area of exit orifice and C_c is the coefficient of vena contracta accounting for a contraction of exit jet relative to the exit orifice (Saleta et al., 2005). In this regard, it was recognized that the fractal properties of the pore network have a sturdy impact on the wetting phenomena (Deinert et al., 2005, 2008). Consequently, the Torricelli's discharge rate may be dependent on the fractal attributes of the pore network. However, possible effects of the fractal features of pore network on the Torricelli's discharge were not studied yet.

The question we address in this work is how the self-similarity of pore network impacts on the Torricelli's discharge from porous reservoirs. In this regard, we first argue that that the porous reservoir will discharge in the Torricelli's regime if, and only if, the volumetric discharge rate admitted by the Bernoulli's principle is smaller than the volumetric flow rate governed by the Darcy's law. Accordingly, in this work, we carried out a comparative study of water discharges from a hollow container and from the same container with the fractal (inverse Menger sponge) and Euclidean (cubic network of straight square channels) filters having the same overall porosity and sufficiently larger absolute permeabilities. Experiments were performed with two different experimental setups. In both cases, the squared outflow velocities are found to be linear functions of water table elevation above the exit orifice. Furthermore, we found that porous filters placed above the exit duct have no effect on the discharge rate. However, when the exit duct is connected to the one of the lateral exits of the pore network, the Torricelli's discharge from the inverse Menger sponge is faster than from the container with the Euclidean filter, but slower than from the hollow container. A decrease of the outflow velocity from the container with filter can be attributed to frictional loss of mechanical energy of flowing water and hydraulic head reduction due to capillary forces in the pore network. Accordingly, we found that the frictional loss of mechanical energy of flowing water in the Menger sponge is found to be smaller than in the cubic network of square channels, while the hydraulic head reduction is found to be the same for both filters.

The frictional loss of mechanical energy of water flowing along the path of least resistance is proportional to ratio $r = S_w/V_w^{2/3}$, where S_w is to the flow wetted area and V_w is the volume of flowing water. In this regard, we argue that the path of least resistance in the scale invariant pore network is characterized by smaller ratio r then the path of least resistance in than in the translationally invariant network of square channels, despite that the total areas of pore-matrix interfaces in the fractal and Euclidean filters are approximately equal.

2. Theoretical background

A laminar flow of a Newtonian fluid is expected to obey both the Bernoulli's principle and the Hagen-Poiseuille law (Sutera, 1993; Synolakis and Badeer, 1989). Accordingly, the fluid flow velocity is determined by a complex interplay of related phenomena (Fiorillo, 2011, 2012; Huppert et al., 2013). In the case of an ideal fluid, the outflow velocity from a hollow container through a short duct is given by the classical Torricelli equation

$$u = \sqrt{\frac{2gh}{1-(s_O/s_C)^2}} \tag{2}$$

where g is the gravity acceleration constant, s_C is the cross-sectional area of the container, and s_O is the cross-sectional area of exit orifice (Malcherek, 2016). The outflow velocity of a Newtonian fluid is always smaller than predicted by Eq. (2) due to frictional loss of mechanical energy of the fluid flowing through the exit duct and hydraulic head reduction due capillary force in the exit duct. These factors can be accounted for via a phenomenological modification of the Bernoulli's equation (Luque-Escamilla, 2009; Saleta et al., 2005). Alternatively, it was suggested that the gravity-driven discharge is governed by the fluid momentum balance (Ferro and Aydin, 2018; Malcherek, 2016). In both cases, the equation for the outflow velocity can be presented in the form.

$$u = C_m\sqrt{2g(h-h_c)} \tag{3}$$

if $s_O << s_C$, where h_c is the hydraulic head reduction due to the capillary force in the exit duct, while the empirical coefficient C_m accounts for either the frictional loss of mechanical energy of water flowing through exit duct (Saleta et al., 2005), or the efflux of fluid momentum due to inhomogeneous velocity distribution over the cross section of the exit duct (Malcherek, 2016). Taking this as a theoretical background, one may expect that, if the wetting phenomena in the pore network are relevant, the Torricelli's discharge from the porous reservoir will also obey the modified Torricelli's Eq. (3), but with the fitting parameters C_m and h_c accounting for the frictional loss of mechanical energy (or momentum efflux) and hydraulic head reduction in the pore network, as well as in the exit duct.

On the other hand, the Darcy's law states that the volumetric flow rate from the porous reservoir is linearly proportional to the hydraulic head (Brown, 2002). Accordingly, in the Darcy's regime of gravity-driven discharge, the outflow velocity $u = Q/C_c s_O$ depends on the hydraulic head as

$$u = \frac{kg}{\mu}\left(\frac{s_C}{s_O}\right)(h-h_c) \tag{4}$$

whenever the water table is above the top of porous filter.

From Eqs. (3) and (4) it follows that the gravity-driven discharge from the container with the porous filter can be characterized by the dimensionless number

$$R = \frac{k^2 g(h-h_c)}{2\mu^2 L_0^2 C_m^2} \tag{5}$$

where L_0 is the filter height. It is a straightforward matter to understand that, if $R < (s_O/s_C)^2$, the gravity-driven discharge will be governed by the Darcy's law. Conversely, if $R >> (s_O/s_C)^2$, the gravity-driven discharge will be controlled by the Bernoulli's principle.

3. Experiments

3.1. Models of porous reservoirs

The Menger sponge is one of the most popular models of scale invariant porous media (Aarão-Reis et al., 2018; Adler, 1996; Cihan et al., 2009). The pore network in the inverse Menger sponge is the Menger sponge itself. Accordingly, in this work, we use the inverse Menger sponge as a filter with the scale invariant pore network. It is constructed by a simple iterative procedure as follows. The initial cube is divided into $3^3 = 27$ identical cubic pieces and seven pieces at the body and face centers are extracted. At the next iteration, the same procedure is applied to 20 remaining pieces. A pre-fractal inverse Menger sponge obtained after n iteration steps contains a self-similar network of interconnected channels with different geometry and cross-sectional