Accepted Manuscript

Long time unconditional stability of a two-level hybrid method for nonstationary incompressible Navier–Stokes equations

Eric Ngondiep

PII: S0377-0427(18)30294-2
DOI: https://doi.org/10.1016/j.cam.2018.05.023
Reference: CAM 11688

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 4 September 2015
Revised date : 3 January 2018

Long time unconditional stability of a two-level hybrid method for nonstationary incompressible Navier-Stokes equations

Eric Ngondiep *

Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, 90950 Riyadh, Kingdom of Saudi Arabia

Hydrological Research Centre, Institute for Geological and Mining Research, 4110 Yaounde-Cameroon

Abstract. This paper presents a two-level MacCormack rapid solver method for solving a 2D time dependent Navier-Stokes problem that models an incompressible fluid flow. A decoupling approach based on the coupling term via spatial extrapolation is proposed for devising decoupled marching algorithms for the nonstationary incompressible model. In this hybrid method, an explicit MacCormack scheme provides a H^1-optimal velocity approximation and a L^2-optimal pressure approximation by solving a nonlinear Navier-Stokes problem on a coarse grid with mesh size H, while an implicit Crank-Nicolson algorithm consists in dealing with the fully discrete linear generalized Stokes problem on a fine grid with mesh size $h << H$. The theoretical result suggests that our method is unconditionally stable over long time intervals. While the numerical evidences both confirm the theoretic analysis and show that the algorithm is convergent, the tests also suggest that our method is both cheaper and faster than the two-level finite element Galerkin scheme.

Keywords: Navier-Stokes equations, explicit MacCormack algorithm, Crank-Nicolson scheme, a two-level MCRS method, stability analysis

AMS Subject Classification (MSC). 65M10, 65M05.

1 Introduction and motivation

The explicit MacCormack algorithm is a suitable method for solving both steady and unsteady flows at moderate to low Reynolds numbers. Two-level finite element Galerkin (FEG) method is an efficient numerical scheme used by several authors [39, 37, 22, 1, 18, 13, 34] to solve nonlinear partial differential equations (PDEs), such as, steady semi-linear elliptic equations and both stationary and nonstationary Navier-Stokes equations while two-grid methods are satisfactory approaches for solving multi-modelling problems [29, 23, 40, 4, 20]. Examples of coupled multi-model applications include viscous-inviscid flows [5], compressible-incompressible fluids [6], turbulent-laminar flows [11], viscous-porous media flows [12, 28] and inertial confinement fusion (ICF) with high ratio of density and temperature [36].

In reality, the MacCormack algorithm provides good resolution at discontinuities and the best resolution of discontinuities occurs when the difference in the predictor is in the direction of the propagation of the discontinuity. This method has been widely used to solve nonlinear PDEs (for examples: Euler equations and Navier-Stokes problems for laminar flow). For multidimensional problems, a time-split version of the MacCormack method has been developed and deeply studied. However, this scheme is not a satisfactory approach for solving high Reynolds numbers flow, where the viscous regions becomes very thin [2], p. 631-632. In fact, the unsteady compressible Navier-Stokes equations are a mixed set of hyperbolic-parabolic equations while the unsteady incompressible Navier-Stokes problems are a mixed set of elliptic-parabolic equations. As a consequence, different numerical schemes have been used in the past to solve the Navier-Stokes equations in the compressible and incompressible flow regime. More recently, methods have been developed to efficiently

*Tel.: +966506048689. E-mail addresses: ericngondiep@gmail.com or ngondieperic@rocketmail.com (Eric Ngondiep).