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LINEAR $*$-DERIVATIONS ON JB^*-ALGEBRAS [1]

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Abstract It is shown that for a derivation

$$f(x_1 \circ \cdots \circ x_{j-1} \circ x_j \circ x_{j+1} \circ \cdots \circ x_k) = \sum_{j=1}^{k} x_1 \circ \cdots \circ x_{j-1} \circ x_{j+1} \circ \cdots \circ x_k \circ f(x_j)$$

on a JB^*-algebra $\mathcal{B}$, there exists a unique $\mathbb{C}$-linear $*$-derivation $D : \mathcal{B} \to \mathcal{B}$ near the derivation.

Key words linear $*$-derivation, JB^*-algebra, functional equation, stability

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1 Introduction

Our knowledge concerning the continuity properties of epimorphisms on Banach algebras, Jordan-Banach algebras, and, more generally, nonassociative complete normed algebras, is now fairly complete and satisfactory (see [7] and [8]). A basic continuity problem consists in determining algebraic conditions on a Banach algebra A which ensure that derivations on A are continuous. In 1996, Villena[8] proved that derivations on semisimple Jordan-Banach algebras are continuous.

Borelli[1] proved the Hyers-Ulam stability problem of the functional equation

$$D(xy) = xD(y) + yD(x)$$

on the interval $(0, 1]$, which is called a derivation, and Tabor[6] investigated the Hyers–Ulam stability problem of the functional equation for Banach space-valued functions and obtained the following result: Let X be a Banach space with norm $\|\cdot\|$ and let $f : (0, 1] \to X$ be a mapping and $\theta > 0$. Suppose that

$$\|f(xy) - xf(y) - yf(x)\| \le \theta$$

for all $x, y \in (0, 1]$. Then there exists a derivation $D : (0, 1] \to X$ such that

$$\|f(x) - D(x)\| \le 4e\theta$$

for all $x \in (0, 1]$.

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Let E_1 and E_2 be Banach spaces with norms $\|\cdot\|$ and $\|\cdot\|$, respectively. Consider $f : E_1 \to E_2$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in E_1$. Assume that

[redacted]

for all $x, y \in E_1$. Rassias[5] showed that there exists a unique $\mathbb{R}$-linear mapping $T : E_1 \to E_2$ such that

$$\|f(x) - T(x)\| \leqslant \frac{2\theta}{2 - 2^p}||x||^p$$

for all $x \in E_1$. Găvruta[2] generalized the Rassias' result, and Park[4] applied the Găvruta's result to linear functional equations in Banach modules over a C^*-algebra.

Throughout this paper, let $\mathcal{B}$ be a JB^*-algebra with norm $\|\cdot\|$, and k an integer greater than 1.

In this paper, we prove that for a derivation

$$f(x_1 \circ \cdots \circ x_{j-1} \circ x_j \circ x_{j+1} \circ \cdots \circ x_k) = \sum_{j=1}^{k} x_1 \circ \cdots \circ x_{j-1} \circ x_{j+1} \circ \cdots \circ x_k \circ f(x_j)$$

on a JB^*-algebra $\mathcal{B}$, there exists a unique $\mathbb{C}$-linear $*$-derivation $D : \mathcal{B} \to \mathcal{B}$ near the derivation.

2 Stability of Linear $*$-Derivations on JB^*-Algebras

The original motivation to introduce the class of nonassociative algebras known as Jordan algebras came from quantum mechanics (see [7]). Let $\mathcal{H}$ be a complex Hilbert space, regarded as the "state space" of a quantum mechanical system. Let $\mathcal{L}(\mathcal{H})$ be the real vector space of all bounded self-adjoint linear operators on $\mathcal{H}$, interpreted as the (bounded) observables of the system. In 1932, Jordan observed that $\mathcal{L}(\mathcal{H})$ is a (nonassociative) algebra via the anticommutator product $x \circ y := \frac{xy+yx}{2}$. A commutative algebra X with product $x \circ y$ (not necessarily given by an anticommutator) is called a Jordan algebra if $x^2 \circ (x \circ y) = x \circ (x^2 \circ y)$ holds.