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Analytical parametrizations of droplet collision efficiency on cylinders – A review study

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ABSTRACT

Within the scope of this work several major ice accretion parameterizations have been investigated, staring from the original Langmuir and Blodgett work 1946) on the water droplet trajectories, up to and including the Finstad et al. model (1988a) of overall collision efficiency, which is part of the current governing ISO 12494 standard (2001), thus covering a timeframe of several decades of investigations in icing modeling. This paper provides a general and mathematical review of those parameterizations, includes necessary formulae for calculations of the droplet overall collision efficiency, starting with the trajectory evaluation, and discuses underlying assumptions and approximations made by respective authors. This discussion might be of interest to icing modelers who wish to obtain more general understanding of icing modeling. As an application example, two experimental datasets have also been used for the droplet overall collision efficiency calculations and comparison. These experiments span large amount of operating conditions, thus covering significant range of the droplet inertia parameter range, *K*, and the overall collision efficiency, *E*, values which should cover majority of possible icing conditions. The results show that for higher values of the droplet inertia parameter (*K*), the monodisperse distribution yields good agreement with the experimental values, however, with gradual decrease in values of droplet's inertia parameter, the MVD approximation tends to underestimate the overall collision efficiency when compared with the experimental and spectrum-averaged values. Moreover, for very low values of *K* and *E*, roughly corresponding to the limits provided in ISO 12494, the MVD approximation tends to underestimate the overall collision efficiency significantly. For those cases the recalculation of droplet trajectories using full spectrum is recommended. If actual droplet distribution spectrum is not available, it is recommended to carry out the analysis using the Langmuir distributions, such as widely used 'Langmuir D' distribution (Wright, 2008), (Bidwell, 2012), (Papadakis et al., 2007).

1. Introduction

Atmospheric icing of structures, is a hazardous phenomenon which may lead to undesirable effects. To properly estimate the potential hazards of atmospheric icing, a good understating of the ice accretion process is needed. Presently, the aggregated knowledge on the modeling of atmospheric icing and its effects is governed by ISO standard, ISO 12494 *"Atmospheric Icing of Structures"*. Most importantly, the main equation in the icing modeling, which describes the rate of icing per unit time is given as (ISO, 2001):

$$\frac{\mathrm{dm}}{\mathrm{dt}} = \alpha_1\alpha_2\alpha_3 \mathrm{wAv} \tag{1}$$

In this equation, otherwise known as the "Makkonen model" (2000), *A* is the cross-sectional area of the object (*with respect to the direction of the particle velocity vector v*), α_1 (also referred as *E* in literature) is the collision efficiency, α_2 is the sticking efficiency, α_3 is the accretion efficiency. The correction factors α_1, α_2 and α_3 represent different processes that may reduce dm/dt from its maximum value *wAv*. These correction factors vary between 0 and 1.Factor α_1 represents the efficiency of collision of the droplets, i.e. is the ratio of the flux density of the droplets that hit the object to the maximum flux density, which is a product of the mass concentration of the droplets, *w*, and the velocity, *v*, of the droplets with respect to the object.

Consequently, the collision efficiency α_1 is reduced from one, because small droplets tend to follow the air streamlines and may be deflected from their path towards the object, as shown in Fig. 1.

In the broadest case of a given fluid flow, the "behavior" of water droplets can be explained using the definition of the Stokes number:

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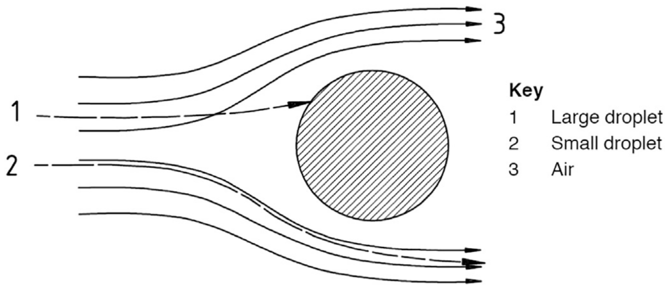

Fig. 1. Air streamlines & droplet trajectories around a cylindrical object (ISO, 2001).

$$Stk = \frac{t_0 u}{L} \tag{2}$$

where L is the characteristic length of the obstacle and t_0 is the relaxation time of the particle, which describes its exponential velocity decay due to influence of drag and it is defined as:

$$t_0 = \frac{\rho_p d_p^2}{18\mu_f} \tag{3}$$

in which ρ_p is the particles density, d_p is the particle's diameter and μ_f is the absolute viscosity of the fluid. A particle with a low Stokes number follows fluid streamlines (*perfect advection*), while a particle with a large Stokes number is dominated by its inertia and continues along its initial trajectory, thus colliding with the object. As it can be seen from Eqs. (2) and (3), larger particles, or those moving at higher velocities, will have higher Stokes number and thus – higher possibility of collision with the object, hence defining physical meaning of the collision efficiency.

However, in reality, the behavior of the droplet in actual flow is much more complicated than in this simplistic case, and the collision efficiency cannot simply be explained using just the definition of Stokes number, thus requiring the use of some sort of analytical and/or empirical formulations in order to calculate the overall collision efficiency. Presently, the overall collision efficiency formulation by Finstad et al. (1988a) is used in the ISO 12494 for calculation of α_1, which is itself based on the earlier parameterization by Langmuir and Blodgett (1946).

While Finstad et al. model is the standard model in icing studies, based on the experimental results of (Makkonen and Stallabrass, 1987), arguments and comparison provided by (Finstad et al., 1988a), and extensive work on "standard" icing model by (Makkonen, 2000), in which the Finstad et al. parametrization is one of the core concepts, which ultimately led to its inclusion in the governing ISO 12494 standard (ISO, 2001), the overview of other historical parameterizations, developed prior to it, might be useful for icing modelers, as majority of those models are based on similar concepts and share core assumptions, applications and limitations.

Broader understanding of those historical parameterizations as well as current parameterization of Finstad et al. might be useful in conducting experimental, numerical and analytical analyses, especially, when there is a need of modeling of the ice accretion in extreme cases, close to the limits of applicability, as given in the ISO 12494 (ISO, 2001) and therefore, the review study of said parameterizations is the main scope of this work. The analytical parameterizations being investigated within the scope of present study are the original Langmuir and Blodgett parameterization (Langmuir and Blodgett, 1946), as well as parameterizations derived by Cansdale and McNaughtan (1977), Stallabrass (1980), Lozowski et al. (1983a), Makkonen (1984), Finstad and Karen (1986) and its'presentversion by Finstad et al. (1988a).

2. Analytical parameterizations of droplet collision efficiency

The purpose of this subsection is to provide a brief overview of the droplet analytical collision efficiency parameterizations, which are within the scope of this study. Each model will be described briefly, in order to provide the general overview, such as, when the model in question was developed, what considerations the respective authors have been using, for what applications the model has been applied and what are the unique characteristics of it, etc. The proper references are provided in each respective paragraph, however, for brevity, the specific equations will be given later.

Langmuir and Blodgett (LB) parameterization (1946). The Langmuir and Blodgett research (1946) was mostly aimed at estimating the water droplet trajectories moving past infinitely long circular cylinder for cases, where Stoke's law is not applicable. Langmuir and Blodgett used a General Electric developed analogue computer, called Differential Analyzer, to obtain the results for 61 droplet trajectories for the flow around cylinders, ribbons and spheres.

The Langmuir and Blodgett model is one of the more complete models featuring parameterizations for the overall and the stagnation line collision efficiencies, the maximum impingement angle and the droplet's impact velocity, along with correction of the overall collision efficiencies for low values of the overall collision efficiency and different parameterization scheme for higher values of overall collision efficiency, $E > 0.5$.

Moreover, Langmuir and Blodgett produced a series of plots for droplet inertia parameter and Langmuir parameter, K, and φ respectively which may be used to obtain results graphically. The validation of results for cylinders was done in the original study, and it consisted of comparison with experimental data from Mt. Washington Observatory, obtained by few rotating cylinders, exposed to icing at various conditions (Langmuir and Blodgett, 1946), in addition to some experimental data, obtained by aircraft flying at 200 mph.

Lozowski et al. parameterization (1979). This parameterization is a part of the model, originally developed in 1979 by Lozowski et al. (1979), and published in 1983 (Lozowski et al., 1983a) for studying helicopter icing with inclusion of liquid water on the surface, known as the "water runback" in it, due to the steady-state heat balance on the cylinder's surface, calculated using Messinger's thermodynamic model (1953), which is the main innovation of this model.

The parameterization of droplet trajectories is essentially similar to Langmuir and Blodgett approach, however slightly different empirical fit was used in order to avoid usage of Langmuir and Blodgett corrections for different ranges of the overall collision efficiency E, thus attempting to use a single parameterization scheme for the entire range of E. Moreover, the model introduced an empirical formulation for the local collision efficiencies β as function of the impingement angle θ, which allows calculation of the ice shapes, with limitation being constant ice density of $\rho = 890\,\text{kg/m}^3$ being used in their model. The experimental verification of model for cases of ice accretion on cylinders have been conducted by Lozowski et al. (1983b), the verification for aircraft icing have been done independently by Bain and Gayet (1982).

Additionally, in 1977 Cansdale and McNaughtan (1977) developed the icing model for similar applications, again, using slightly re-defined values of original Langmuir and Blodgett parameterization scheme for the droplet collision efficiency, in order to collapse it to single curve for the entire range of E, which also differs from parameterization values those of Lozowski et al. (1983a). It is deemed appropriate to include both parameterizations in this study to observe the differences in droplet collision efficiency values between two similar models, developed roughly at the same time and for similar applications. However, Cansdale and McNaughtan model is more simplistic in its approach and only takes into account the flow near stagnation point.

Stallabrass parameterization (1980). This model was developed for studying icing of fishing trawlers (Stallabrass, 1980). The main difference in this model, when it comes to the droplet collision efficiency parameterization, is an attempt to eliminate the use of multiple curves and the droplet trajectory equations altogether for estimation of the overall collision efficiency, and collapse the parameterization to a