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Physics of the Earth and Planetary Interiors 150 (2005) 45–61

PHYSICS OF THE EARTH AND PLANETARY INTERIORS

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Reciprocity in electromagnetics: application to modelling marine magnetometric resistivity data

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Received 18 July 2003; received in revised form 3 November 2003; accepted 16 August 2004

Abstract

The marine magnetometric resistivity (MMR) can be used to obtain resistivity structure beneath the ocean floor. Because of logistical reasons, many more transmitters than receivers are deployed in a survey. This makes it difficult to carry out 3-D forward modelling of magnetometric resistivity data, since each transmitter source requires a separate solution of Maxwell's equations in order to generate the fields. Two methods are presented to overcome this difficulty. The first is based upon the Lorentz reciprocity theorem. With this theorem, the magnetic field at a receiver, generated by a long vertical electrical bipole, is exactly the same as the normalized electromotive force induced in the transmitter wire generated by an artificial magnetic dipole located at the receiver position. The second is the adjoint method in which the magnetic field can be obtained by solving an adjoint equation with an artificial source at each receiver. We show that these two methods are eventually identical: the artificial source in both methods is a steady current in a loop, and the "measurement" is the voltage along the transmitter wire. However the adjoint algorithm is computationally more efficient and we use it in the 3-D marine MMR forward modelling. We verify the code with a synthetic 3-D example. Use of the reciprocity significantly reduces the computational load, making the practical marine MMR problem tractable.

Keywords: Lorentz reciprocity; Adjoint; Magnetometric resistivity; Electromagnetic induction; Forward modelling; Marine electromagnetics

1. Introduction

The marine magnetometric resistivity (MMR) method is an electromagnetic (EM) exploration method that has been successfully used for investigation of electrical resistivity structures of the seafloor (Chave et al., 1991). The geometry of the system is shown in Fig. 1, which is an adaptation of terrestrial MMR approaches (Edwards et al., 1985). The method essentially involves measurement of magnetic fields associated with man-made, non-inductive (low-frequency or pseudo-DC) current flow energized into the seawa-

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doi:10.1016/j.pepi.2004.08.015

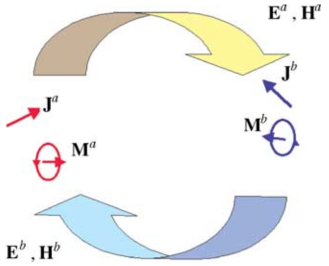

Fig. 1. A schematic illustrating the reciprocity relationship between a transmitter source **a** and a receiver source **b**.

ter and seafloor through two vertically separated electrodes. The magnetic field measured at the ocean bottom magnetometer depends on the total current entering the seafloor through an Ampère circuit centered on the source and passing through the receiver. In the presence of a layered seafloor, the magnetic field generated by the bipole source possesses an azimuthal symmetry, and the bulk resistivity of the seafloor can be estimated from the amplitude of the magnetic field, given that the current transmitted is known. For general 3-D resistivity structures beneath the seafloor we have to resort to a numerical algorithm (Chen et al., 2002) for both forward modelling and inversion.

A unique characteristic of the marine MMR lies in the reality that there are many more transmitter sources than receivers. In the Juan de Fuca Ridge example (Evans et al., 1998), 34 transmitter sources were deployed while there were only three receivers. This is in contrast to the terrestrial situation, in which only a few (sometimes one!) transmitters are deployed. In a recent MMR experiment in the Eastern Pacific Rise (Evans and Webb, 2002), more than 200 source bipoles were used and 10 three-component magnetometers were placed on the seafloor. From a numerical perspective, this makes it difficult to carry out 3-D forward modelling and inversion of marine MMR data, since each transmitter source requires a separate solution of Maxwell's equations in order to generate the fields.

This difficulty motivated us first to consider applying the Lorentz reciprocity theorem to reduce the heavy computational load. According to the reciprocity theorem, in its simplest sense, a response of a system to a source is unchanged when source and receiver are interchanged (Harrington, 1961). By response, we do not necessarily mean the electric or magnetic field. In Parasnis's tutorial (1988), a response might be a voltage or complex electromotive force (emf) developed in a receiver. In principle, invoking reciprocity can greatly reduce the computations. If the number of receivers is N_{rx}, and three components of magnetic field are measured, we only need to put $3N_{rx}$ artificial magnetic dipole (or current loop) sources at N_{rx} receiver locations (*x*-, *y*-, and *z*-oriented in turn at one receiver location), regardless of the original number of transmitters. In the EPR experiment mentioned above, instead of working with 200 transmitters in forward modelling, we need only 30 artificial sources, or, 20 if only *x*- and *y*-components are used. This reduces the forward modelling computations by an order of magnitude.

The adjoint method, originally defined by Lagrange, has since been thoroughly substantiated and broadly applied in solving many problems (e.g., Marchuk et al., 1996). A common usage in EM investigation is to use the adjoint method to compute the sensitivity (e.g., Weidelt, 1975; Madden, 1990; McGillivray et al., 1994). In this paper, we use the adjoint method to reformulate our 3-D MMR forward modelling. Interestingly, we can find similarities in terms of computational load and "physical" meaning between the Lorentz reciprocity and the adjoint method. As a matter of fact, the adjoint field is closely related to electromagnetic migration field, which was introduced by Zhdanov et al. Interested readers can refer to their work (e.g. Zhdanov and Frenkel, 1983; Zhdanov et al., 1996; Zhdanov, 2002), and to time reversal methods (Borcea et al., 2002). In fact, migration techniques and their derivation relay on the properties of the adjoint equation. Our intention here is to specifically analyze the adjoint of the MMR experiment and its use.

In our opinion the relationship between Lorentz reciprocity and the adjoint method deserves further investigation. Although Lorentz reciprocity is physically realizable (we can set out physical instruments to generate and measure the fields), and the adjoint is a mathematical tool, we find that the underlying physical ideas are identical and the two approaches produce the same numerical solution. For the MMR problem, this intimate relation can be revealed by: (1) investigating the low-frequency characteristic of the electric field due to a magnetic dipole in a general 3-D medium; (2) numerically implementing the adjoint algorithm, in which the artificial source for the adjoint equation is exactly equivalent to a magnetic dipole, but with a frequency of zero; (3) examining the inner product in the adjoint