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Modal properties of cyclically symmetric systems with central components vibrating as three-dimensional rigid bodies

Bin Dong, Robert G. Parker *

Department of Mechanical Engineering, Virginia Tech, Blacksburg, VA 24061, United States

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ABSTRACT

This study investigates the vibration mode structure of general cyclically symmetric systems with central components vibrating as three-dimensional rigid bodies. This work does not rely on the assumptions of the system matrix symmetries; asymmetric inertia matrix, damping, gyroscopic, and circulatory terms can be present. In the equation of motion of a general cyclically symmetric system, the matrix operators are proved to have properties related to the cyclic symmetry. These symmetry-related properties are used to prove the modal properties of general cyclically symmetric systems. Only three types of modes can exist: substructure modes, translational-tilting modes, and rotational-axial modes. Each mode type is characterized by specific central component modal deflections and substructure phase relations. Instead of solving the full eigenvalue problem, all vibration modes and natural frequencies can be obtained by solving smaller eigenvalue problems associated with each type. This computational advantage is dramatic for systems with many substructures or many degrees of freedom per substructure.

1. Introduction

Models for many rotating mechanical systems are cyclically symmetric, including bladed discs [1–3], circular plates [4–6], circular rings [7–9], disc-spindle systems [10,11], planetary gears [12–18], centrifugal pendulum vibration absorbers (CPVA) [19–23], *etc*. In such systems, identical *substructures* (*e.g.*, the planets of planetary gears) are equally spaced circumferentially around the *central components*, such as the sun gear, ring gear, and carrier in planetary gears. As shown in Fig. 1, a typical cyclically symmetric system is invariant to rotations by the spacing angle. Such structural symmetry results in unique modal vibration properties, and knowing these properties aids the analysis of the systems.

Studies of cyclically symmetric systems date back at least 90 years when physicists studied the lattice vibration of one-dimensional mono-atomic linear chains. With the Born-von Karman periodic boundary condition, the symmetry of a one-dimensional Bravais lattice is cyclic [24], and its dispersion relation shows that the frequency passband widths are determined by the coupling strength between substructures (atoms) [25]. Similar studies of wave propagation in substructure chains indicate unique phase relations exist between the substructures in different modes [26–28]. Such phase relations can be characterized by phase indices [10,11] that are analogous to the wave vectors in lattice vibrations [29] and the numbers of nodal diameters in cyclically symmetric continuum vibrations [7,11]. The phase relations or indices can be mathematically explained by diagonalization of the system mass and stiffness matrices that are circulant or block circulant. The basic theory of circulant matrices [30] was applied to the vibration modes of bladed discs by Óttarson [2] and Olson [31]. The eigenvalue problems are

* Corresponding author.
E-mail addresses: bind89@vt.edu (B. Dong), r.parker@vt.edu (R.G. Parker).

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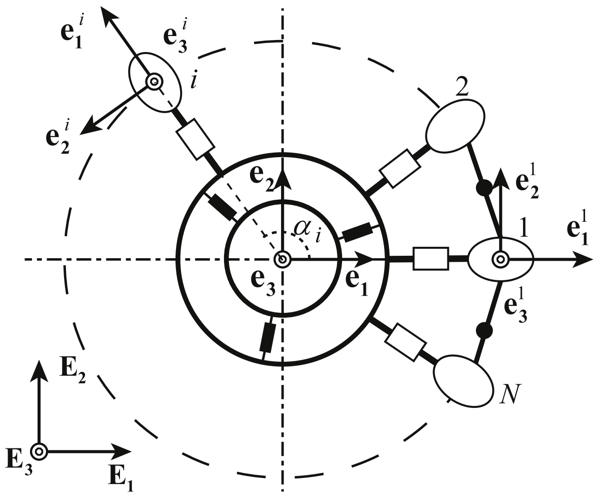

Fig. 1. Cyclically symmetric systems with multiple central components. The circles represent possible central components, and the ovals are substructures. Some substructures are omitted for conciseness. Couplings may exist between any two substructures (represented by solid lines with solid circles), although only the couplings between the nearest neighboring substructures are depicted. Solid lines with hollow rectangles are the couplings between the central components and the substructures. All the central components and the couplings between them (represented by solid rectangles) are axisymmetric along the nominal rotational axis.

significantly reduced in size after unitary transformation. A summary of the theory of circulant matrices is provided in a later work by Olson et al. [32]. The theory is directly applicable to the modeling and analysis of free and forced vibration of cyclically symmetric systems *without* vibrating central components.

The assumption that central components are non-vibrating simplifies the modeling and analysis of cyclically symmetric systems. In many cyclically symmetric systems, however, vibrations of the central components are significant. The central components are circumferentially uniform and axisymmetric along the system's central axis. The full-system matrices are no longer circulant or block circulant when considering the motions of central components. In many past works, the central components are restricted to planar vibration with two translational degrees of freedom and one rotational degree of freedom along the system's central axis. One example of this case is spur planetary gears [12–18], where the sun gear, ring gear and carrier are the central components and the planet gears are the substructures. Another example is CPVA systems [19–22], where the rotor is the central component and the pendulum absorbers are the substructures. Shi and Parker [33] mathematically proved the highly structured modal properties of cyclically symmetric systems assuming the central components have only planar motions, which is a limitation inconsistent with the fact that the substructures can have general three-dimensional geometry and motions.

It remains unknown what modal properties exist when central components are allowed to vibrate as rigid bodies with general six-degree-of-freedom motions, while the substructures can have an arbitrary number of in-plane and out-of-plane degrees of freedom (including general finite element models). For example, the gear mesh interfaces in helical planetary gears generate three-dimensional forces and moments, and the motions of the central components (the sun gear, ring gear, and carrier) involve all six rigid body degrees of freedom. The modal properties summarized in the prior works are not thoroughly applicable. Eritenel and Parker [34] studied the modal properties of helical planetary gears, and similar work was conducted for the three-dimensional CPVA system [23]. Both works showed that some modal properties of cyclically symmetric systems with planar vibrating central components are preserved, while others differ. General cyclically symmetric systems with central components that can vibrate in all six degrees of freedom have not been studied.

The goal of this paper is to mathematically prove the modal properties of generalized cyclically symmetric systems in which the central components have arbitrary three-dimensional motions. The theory generalizes the properties of cyclically symmetric systems with planar vibrating central components in Ref. [33]. Although the properties are derived for linearized discrete systems, they can be extended to linearized continuum and continuum-discrete hybrid systems if the analogy between phase indices and wavenumbers is established.

This paper is organized as follows. In Section 2, the modal properties of cyclically symmetric systems with non-vibrating central components are reviewed, and the basic theory of circulant matrices is distilled from Ref. [32]. The study of cyclically symmetric systems with vibrating central components starts by proving essential properties of the system matrices, which is discussed in Section 3. In Section 4, the modal properties of cyclically symmetric systems with vibrating central components are derived, and we prove that only three categories of modes exist: substructure modes, translational-tilting modes, and rotational-axial modes. Reduced eigenvalue problems for each category are identified, so all eigensolutions can be solved with less computational effort. An example is given to illustrate the properties.