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# Entropy generation in dissipative flow of Williamson fluid between two rotating disks

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ABSTRACT

This communication addresses flow characteristics of Williamson fluid between two rotating disks. Electrically conducting fluid is considered. Total entropy generation rate is calculated through implementation of second law of thermodynamics. The lower and upper disks have different stretching rates and angular velocities for rotation. Characteristics of heat transport are expressed through dissipation, heat generation/absorption and thermal radiation. Von Karman transformations are utilized to convert the dimensional flow expressions into dimensionless form. Convergent series solutions are constructed. Influence of different pertinent variables on velocity, temperature, entropy number, skin friction coefficients and Nusselt numbers are studied. It is observed that the axial and tangential velocities increase for higher Weissenberg number while radial velocity decays. Further entropy number remarkably increases for magnetic, radiation and Brinkman number. Bejan number is less for higher magnetic parameter, stretching parameter, Brinkman number and Weissenberg number.


## 1. Introduction

Flow investigation by stretchable rotating disk has significance in different mechanical and industrial engineering process like medical equipment, spin coating, food processing technology, manufacturing, air cleaning machine, centrifugal pumps, electric power generating system, pumping of liquid metals at high melting point, turbo-machinery and gas turbines. Flow of non-Newtonian fluid, due to rotating disk is very important area of research. Non-Newtonian fluids have many applications in polymer sheet, production of papers, food processing, production of petroleum, physiology etc. Single constitutive equation is not sufficient for description of non-Newtonian fluids. It is for diversity of liquids in nature. Inspired by the above-mentioned applications, initially Karman [1] was presented a mathematical model for flow problem by rotating disk. Recently, flow investigation through stretchable rotating disk has gained much consideration due to their wide applications in different mechanical and industrial engineering process like medical equipment, spin coating, food processing technology, manufacturing, air cleaning machine, centrifugal pumps, electric power generating system, pumping of liquid metals at high melting point, turbo-machinery and gas turbines. Inspired by the above-mentioned applications, initially Karman [1] presented a mathematical model for flow by a rotating disk. Hayat et al. [2] presented flow of second grade liquid due to a rotating disk. Siddiqui et al. [3] examined electrically conducting flow of Burgers liquid between two disks with heat transfer and Hall current. Mustafa and Khan [4] studied MHD nanomaterials flow due to rotating disk. The flow is discussed in the presence of three different types of nanoparticles i.e., copper (Cu), magnetite or iron oxide ($Fe_3O_4$) and silver (Ag). Nonlinear flow expressions are numerically solved with the help of $bvp4c$ method. Their obtained results predict that velocity and temperature show contrast behavior for higher estimations of nanoparticles volume fraction. Hu et al. [5] investigated thermocapillary flow instabilities through counter rotating disks. The basic solutions of temperature and velocity are obtained using pseudo spectral Chebyshev technique. Turkyilmazoglu [6] explored heat transport in flow of nanofluid by a rotating disk. Further MHD three-dimensional stagnation point flow by a rotating stretchable disk is studied by Turkyilmazoglu [7]. Hayat et al. [8] examined non-Fourier heat flux in flow of viscous material between two rotating disks. Flow of single and multi-walls carbon nanotubes in context of multiple slip conditions by a rotating disk is scrutinized by Hayat et al. [9]. Electrically conducting radiative flow of water based CNTs with convective boundary condition between two stretchable rotating disks is analyzed by Jyothi et al. [10]. Khan and Sultan [11]

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**Nomenclature**

| | | | |
|---|---|---|---|
| $r, \vartheta, z$ | cylindrical coordinates | $\sigma^*/k^*$ | Stefan–Boltzmann constant/mean absorption coefficient |
| $u, v, w$ | velocity components | $M$/Pr | Hartmann number/Prandtl number |
| $\rho$ | density | $Q_0$ | heat source/sink coefficient |
| $a_1, a_2$ | stretching rates of lower and upper disk respectively | $Q^*$ | heat generation/absorption parameter |
| $v$ | kinematic viscosity | $R/Ec$ | radiation parameter/Eckert number |
| $p$ | pressure | $A_2$ | stretching parameter of upper disk |
| $\sigma$ | electrical conductivity | $\alpha_1, \alpha_2$ | temperature ratio parameter |
| $k$ | thermal conductivity | $Br$ | Brinkman number |
| $c_p$ | specific heat | $A_1$ | stretching parameter of lower disk |
| $T$ | temperature of fluid | $\Omega$/Re | rotational parameter/Reynolds number |
| $\tilde{h}, \tilde{f}, \tilde{g}$ | axial, radial and tangential velocities | | |
| $\tilde{\theta}$ | temperature | | |
| $We$ | Weissenberg number | | |

elucidated the MHD transport of Williamson fluid due to rotating disk with slip condition.

Entropy physically corresponds to disorder of a system and surrounding. Heat is well recognized as a form of energy. When it occurs then some extra movements appear for example molecular vibration, molecular friction, spin moment, internal displacement of molecules, kinetic energy etc., which is responsible for the loss of heat energy. Thus heat cannot be converted fully into work. This extra movement creates chaos. It is also called measure of chaos. Entropy and entropy production have crucial roles in numerous diverse phenomena like refrigeration, solar energy and energy storage systems. Initially Bejan [12] showed how the entropy production rate can be decreased in simple components for heat transport like heat exchanges with prescribed heat flux distribution, counter flow gas to gas heat exchangers and sensible heat units for energy storage. Ijaz et al. [13] studied entropy generation in flow of Sisko liquid subject to heat generation/absorption. Flow is investigated over a stretched surface. Nonlinear flow expressions are solved for series solutions via homotopy method. The obtained outcomes predict that velocity field diminishes for higher material variable while thermal field increases for larger radiation parameter and Biot number. Vatanmakan et al. [14] explored steam flow in turbine blades with entropy generation and volumetric heating. Numerical simulation is conducted through two phase Eulerian description for steam flow. Khan et al. [15] examined activation energy impact in chemically reactive flow of Casson fluid with entropy generation. Gul et al. [16] studied mixed convection Poiseulle flow of viscoelastic nanofluid with entropy generation. Xie and Jian [17] discussed MHD two-layer electroosmotic flow with entropy generation through micro-parallel channels. Ijaz et al. [18] scrutinized forced convection flow of viscous nanofluid for entropy generation. Huminic and Huminic [19] analyzed heat transfer performances of hybrid nanomaterials with entropy generation in a flattened tube. Farooq et al. [20] examined entropy generation in flow of carbon nanotubes with thermal radiation and mixed convection. Kiyasatfar et al. [21] explored entropy production and flow of power law fluid in circular micro-channels with slip conditions.

Present study investigates entropy generation in flow of Williamson fluid between two rotating disks. Fluid is conducting in presence of constant applied magnetic magnetic field. Total entropy generation rate is calculated by implementation of thermodynamics second law. Features of heat transport are scrutinized through heat generation/absorption, dissipation and thermal radiation. Nonlinear system is solved for series solutions by homotopy analysis method [22–45]. Impacts of different flow parameters on velocity, entropy generation, temperature, skin friction coefficients and Nusselt numbers are discussed graphically. Concluding remarks are arranged.

## 2. Constitutive equations

Here we are analyze steady and incompressible flow of Williamson fluid between two rotating disks. Flow is considered axisymmetric. Entropy generation is examined for additional effects of heat source/sink and thermal radiation. Stretching rate, angular velocity and temperature of lower disk are $a_1, \Omega_1$ and $\widehat{T}_1$ while for upper disk the stretching rate, angular velocity and temperature are denoted by $a_2$, $\Omega_2$ and $\widehat{T}_2$ respectively. We denote $B_0$ as the magnetic strength applied in $z$-direction (see Fig. 1). Governing equations for velocity and temperature are [11]:

$$\frac{\partial \hat{u}}{\partial r} + \frac{\hat{u}}{r} + \frac{\partial \hat{w}}{\partial z} = 0, \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial r} + \hat{w}\frac{\partial \hat{u}}{\partial z} - \frac{\hat{v}^2}{r}\right) + \frac{\partial p}{\partial r} = \frac{\partial \overline{S}_{rr}}{\partial r} + \frac{\partial \overline{S}_{zr}}{\partial z} + \frac{\overline{S}_{rr} - \overline{S}_{\psi\psi}}{r} - \frac{\sigma}{\rho}B_0^2\hat{u}, \tag{2}$$

$$\rho\left(\hat{u}\frac{\partial \hat{v}}{\partial r} + w\frac{\partial \hat{v}}{\partial z} + \frac{\hat{v}\hat{u}}{r}\right) = \frac{\partial \overline{S}_{r\psi}}{\partial r} + \frac{\partial \overline{S}_{z\psi}}{\partial z} + \frac{2\overline{S}_{r\psi}}{r} - \frac{\sigma}{\rho}B_0^2\hat{v}, \tag{3}$$

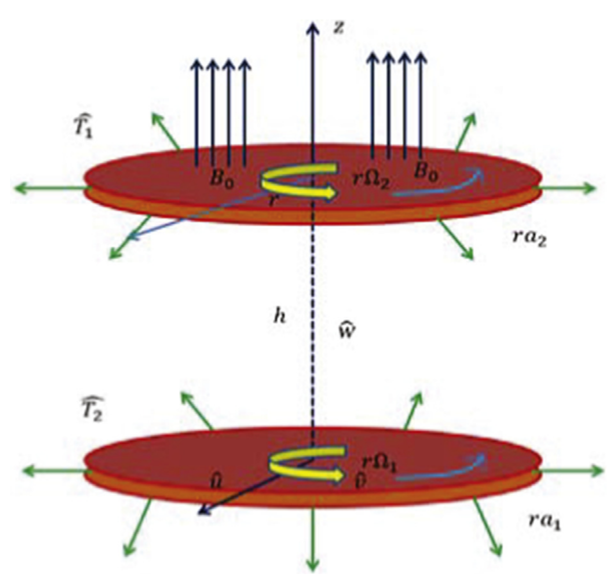


**Fig. 1.** Flow diagram.