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High-order effects of thermal lagging in deformable conductors

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ABSTRACT

The purpose of the present paper is to investigate about the well-posedness question for a thermoelastic model describing the intrinsic coupling between the high-order lagging behavior and the ultrafast deformation. The main motivation behind this study is that the interaction among multiple energy carriers progressively gains significance as the observation scales reduce and has, as a direct consequence, the involvement of high-order terms in the dual-phase-lag constitutive equation for the heat flux. Moreover, the ultrafast deformation can occur in times comparable to the characteristic times in phonon-electron interactions that are due to hot-electron blast and thermal expansion of the lattices. Considering an inhomogeneous and anisotropic linear thermoelastic medium, we are able to prove the uniqueness and continuous dependence results through the use of appropriate integral operators entering into the constitutive equations and Lagrange identities and the time weighted function method, provided very mild assumptions are supposed upon the thermoelastic characteristics.

1. Introduction

The physical foundation and microscale heat-transfer mathematical models, describing the lagging response in times comparable to the phase lags characterizing the microstructural interactions, have been presented in a series of papers by Tzou [1–4]. The refined structure of the lagging response is depicted by means of the high-order effects in correlation with the heat-transfer models in micro/nanoscale (like the systems with multiple energy carriers, including the bioheat transfer and mass interdiffusion) in [3]. We refer, in other words, to the following constitutive equation:

$$q_i(\mathbf{x},t)+\frac{1}{1!}\tau_q\frac{\partial q_i}{\partial t}(\mathbf{x},t)+\frac{1}{2!}\tau_q^2\frac{\partial^2 q_i}{\partial t^2}(\mathbf{x},t)+\cdots+\frac{1}{n!}\tau_q^n\frac{\partial^n q_i}{\partial t^n}(\mathbf{x},t)$$
$$=-k_{ij}(\mathbf{x})\left[T_{,j}(\mathbf{x},t)+\frac{1}{1!}\tau_T\frac{\partial T_{,j}}{\partial t}(\mathbf{x},t)+\frac{1}{2!}\tau_T^2\frac{\partial^2 T_{,j}}{\partial t^2}(\mathbf{x},t)+\cdots+\frac{1}{m!}\tau_T^m\frac{\partial^m T_{,j}}{\partial t^m}(\mathbf{x},t)\right]. \quad (1)$$

Here, q_i and $T_{,i}$ represent the heat flux vector and the temperature variation gradient, τ_q and τ_T are the positive delay times involved in the dual-phase-lag heat conduction model, k_{ij} is the positive definite conductivity tensor and n, m are the Taylor series expansion orders. The dual-phase-lag heat conduction theory has been over the years the subject of a really intense and extensive research activity (the reader can refer for instance to the series of papers [5–25]).

The lagging behavior as described by constitutive Eq. (1) is in well accord with the novel experiments for ultrafast pulse-laser heating on metal films made by Brorson et al. [26] and Qiu et al. [27] (see e.g. [4], pp. 198–200). Bertman and Sandiford [28] have shown that the additional delay in time between the heat flux vector and the temperature gradient is due to the finite time required for activating the low-temperature molecules to transport heat. Antaki [29] used the dual phase lag model of heat conduction to offer a new interpretation for experimental evidence of non-Fourier conduction in processed meat. It was shown there that the dual-phase-lag model combines the wave features of hyperbolic conduction with a diffusion-like feature of the evidence not captured by the hyperbolic case and it is outlined that it accounts for the heterogeneous nature of the meat that is not accommodated by the classical Fourier model.

It is worth to outline the recent paper by Kovacs and Van [30] where a critical survey of the dual-phase-lag concept is presented. The differential dual-phase-lag model of heat conduction has been

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interpreted within the framework of non-equilibrium thermodynamics and it has been concluded that this interpretation removes the most important paradoxes. A series of papers by Rukolaine [31,32], Wang et al. [33] and Zhukovsky [34,35] claim the negative temperature paradox on the dual-phase-lag models. Concerning this, Kovacs and Van [30] write "An inconsistent, unphysical behavior is shown, and the temperature achieves the negative domain again. This is actually an old problem, also mentioned several times for the Maxwell-Cattaneo-Vernotte equation. In fact it is not paradoxical, if we are considering relative temperatures as in the linearized ...equations."

Tzou in [4], Chapters 10 and 12, outlines the extension of the dual-phase-lag model involving the high-order effects in thermal lagging to a wide variety of heat transfer problems from microscale to nanoscale levels. As examples of high-order effects of thermal lagging we call the modeling bioheat transfer phenomena between tissues and blood during the nonequilibrium processes and the complex heat transport processes in biological systems involving numbers of energy carriers and the microstructural interaction effects and related transient processes.

It is worth noting that Chiriţă et al. [36] considered the question of compatibility of the constitutive Eq. (1) with the Second Law of thermodynamics and it has been shown there that for $n \geqslant 5$ or $m \geqslant 5$ the corresponding models lead to some instable mechanical systems; instead, when the approximation orders are lower than or equal to four, then the corresponding models can be compatible with the thermodynamics, provided some appropriate restrictions are assumed upon the delay times.

On the other side, the well-posedness issue for various models of thermoelasticity has been intensively investigated from the mathematical and thermomechanical viewpoints, having in mind its usefulness in the study of several practical applications. We recall here only the studies developed by Brun [37], Knops and Payne [38] and Rionero and Chiriţă [39] in the classical linear thermoelasticity, but we have to mention also the numerous papers on the subject in the Encyclopedia of Thermal Stresses edited by Hetnarski [40] and the review articles by Knops and Wilkes [41] and by Knops and Quintanilla [42]. The qualitative properties established there not only explain how solutions behave but also provide firm foundations for numerical procedures. Knowing whether or not a solution is unique is important for numerical evaluations or for the completeness of constructed by semi-inverse or similar methods. The continuous data dependence, for its part, is also of great practical and numerical importance since the physical measurements introduce unavoidable errors, and it is important to be aware that these small errors influence the real solution in little measure.

In this paper, under inhomogeneous and anisotropic assumptions, we will consider one of the probably most general constitutive equations for the heat flux vector

$$a_0 q_i(\mathbf{x},t) + a_1 \frac{\partial q_i}{\partial t}(\mathbf{x},t) + a_2 \frac{\partial^2 q_i}{\partial t^2}(\mathbf{x},t) + \cdots + a_n \frac{\partial^n q_i}{\partial t^n}(\mathbf{x},t)$$
$$= -k_{ij}(\mathbf{x})\left[b_0 T_{,j}(\mathbf{x},t) + b_1 \frac{\partial T_{,j}}{\partial t}(\mathbf{x},t) + b_2 \frac{\partial^2 T_{,j}}{\partial t^2}(\mathbf{x},t) + \cdots + b_m \frac{\partial^m T_{,j}}{\partial t^m}(\mathbf{x},t)\right], \tag{2}$$

where $a_0, a_1, \ldots, a_n$ and $b_0, b_1, \ldots, b_m$ are prescribed real parameters. It generalizes the constitutive Eq. (1) and the coincidence with it exists clearly if we set

$$a_0 = 1, \quad a_1 = \frac{1}{1!}\tau_q, \ldots, \quad a_n = \frac{1}{n!}\tau_q^n,$$
$$b_0 = 1, \quad b_1 = \frac{1}{1!}\tau_T, \ldots, \quad b_m = \frac{1}{m!}\tau_T^m. \tag{3}$$

In the literature on the heat transport in rigid conductors, the constitutive Eq. (2) is analyzed (see, for example, Chiriţă [43]) in order to prove the uniqueness and well-posedness of a process driven by such constitutive equation. However, the assumption of a rigid conductor is difficult to hold in microscale/nanoscale of heat transfer because the deformation is of the order comparable with the physical scale of interest. So the thermomechanical coupling is highly recommended in deformable conductors, in times when lattice heating becomes pronounced after the thermalization between electrons and phonon/lattices (see, e.g. Tzou [4], Chapter 11). It becomes necessary as the thermally induced mechanical strain rate is sufficiently high.

As remarked by Tzou [4], Chapter 12, "Mathematically, the coupled partial differential equations describing energy and momentum transport through the deformable and conducting medium must be solved simultaneously for temperature and displacement. Generally speaking, this approach involves the solutions for three coupled partial differential equations, two for the temperature and the heat flux vector and another for the displacement vector. Counting all the components in the heat-flux and the displacement vectors, a total of seven coupled partial differential equations could be involved in resolving the energy and momentum transport in deformable conductors. The mathematical details involved are nontrivial. Particular solutions characterizing the fundamental behavior of thermomechanical coupling may be found in special cases, exemplified by the thermal stresses around a rapidly moving heat source accommodating the fast-transient effect of thermal inertia (Tzou, 1989a, b; 1992c), but a general solution is not guaranteed, especially for problems involving finite boundaries."

What stated above is therefore certainly sufficient to consider of great interest the investigation at issue about the uniqueness and the well-posedness questions for the coupling thermomechanical models which rise towards higher effects of thermal lagging. In our analysis we succeed to develop a mathematical technique based on the Lagrange identity and the time weighted function methods which is able in getting the following achievements:

(i) treating simultaneous of the coupled partial differential equations describing energy and momentum transport through the deformable and conducting medium, avoiding in this way the high-order regularity required by complicated conversion between the displacement vector and the temperature variation;
(ii) treating all the mixed boundary mathematical formulations in terms of both displacement vector and temperature variation, as well as for the stress tensor and heat flux vector in the boundary conditions;
(iii) obtaining the results under the mildest assumptions upon the thermoelastic characteristics;
(iv) it is worth noting that our choices of (n,m) orders are able to cover different modes of heat transfer: (a) when $n = m$ we have a diffusive behavior; while when $n = m \pm 1$ we have a wave-like behavior.

The manuscript is organized as follows: in Section 2 the appropriate mixed initial-boundary value problem is formulated and, on the basis of the two operators involved into the constitutive Eq. (2), the modified mixed initial-boundary value problems are defined. Section 3 is devoted to establish a Lagrange type identity which is further combined with the time weighted function technique. In the Section 4 we provide two possible measures of the gradient of the temperature variation that will be useful to show the uniqueness results presented in the Section 5. Finally, the Section 6 is devoted to prove the continuous dependence results, provided the solutions are assumed in an appropriate class of functions.