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Influence of capillary pressure gradient on connectivity of flow through a porous medium

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ABSTRACT

We study the nonlinear development of perturbations of the phase transition surface in the Hele-Shaw cell and a porous medium with a change of capillary pressure gradient along the interface. A bounded water-saturated region surrounded by humid air is explored. Capillary pressure at the interface causes deformation and movement of the water-saturated region. An approximate analytical solution using the method of characteristics is derived and compared with numerical simulation results. It is shown that the evolution of the deforming region can lead to its breakup into disconnected parts. Numerical experiments also show that the deformation of closely spaced separate regions can lead to their fusion. Such the reconfiguration of the interfaces can cause significant changes in the flow properties. Fragmentation during the motion and evaporation of groundwater leads to the formation of isolated inclusions of the substance carried by the water flow.

1. Introduction

In nature and in technological processes, there are many examples of fragmentation of filtration flows. A classic example of a change in the connectivity of the filtration flow occurs in the operation of oil deposits. When oil is displaced by water, the fragmentation of the flow leads to the formation of residual oil. Moreover, along with the influence of the viscosity causing the instability of the interface, an important role is played by capillary forces acting at the oil-water front [1–3]. In the course of motion of the groundwater containing dissolved admixture or suspended particles, the fragmentation of flow leads to the formation of motionless isolated water-saturated regions and, as a result of the water evaporation, inclusions of the solid phase are formed [4,5]. Another example is the operation of an inkjet printer when individual drops of ink are absorbed by a multilayer sheet of paper [6]. The topology transitions in the interface occurs at the nonlinear stage of perturbation development and radically changes the properties of the flow.

In recent works [7,8,10], the stability of phase transition interfaces with a constant capillary pressure acting on the water-humid air interface was investigated. If the capillary pressure is variable, as in the case of inhomogeneous porous media, then the stability conditions of the flow undergo radical changes and the interface can be stabilized or destabilized [11,12]. This heterogeneity can be caused by changes in porosity, permeability, as well as changes in the composition and physicochemical properties of soils and porous rock.

The nonlinear stage of perturbation development is characterized by the formation of fingers, which on a large scale of time undergo complex deformations [13]. The evolution of nonlinear perturbations (fingers) is a stochastic process, and the difficulties arising in its study are sometimes insurmountable. For the detailed numerical simulation of a single finger or a group of fingers, the initial shape of the perturbation should be chosen. The shape of the finger can be taken from the calculations of the unstable interface destruction [14,15] or it can be chosen randomly. In the latter case, it will be difficult to find an analytical solution, which is very important for testing the results of numerical simulations. Therefore, in this paper we study the deformation of water-saturated regions having a regular simple geometric shape to understand the dynamics of topological transitions.

Topological transitions such as pinchoff and reconnection of interfaces are fundamental features of multicomponent fluid flows. Possible topology changes in the interface separating two immiscible fluids in the Hele-Shaw cell were investigated in paper [16]. The Saffman-Taylor approximation was used in [16] and the

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Nomenclature

F_c	the dimensionless function of dependence of the capillary pressure on the coordinate z
G	dimensionless acceleration of gravity
g	acceleration of gravity [m s^{-2}]
H	dimensionless height of a rectangle [m]
h	dimensionless X-coordinate of the interface
h_0	dimensionless half-width of a rectangle
h_m	dimensionless minimum value of h
k	permeability [m^2]
l	half-width of the layer in which the capillary pressure varies [m]
P	dimensionless pressure
P_c	dimensionless capillary pressure
p	pressure [Pa]
p_c	capillary pressure [Pa]
p_{c1}	capillary pressure at $z < -l$ [Pa]
p_{c2}	capillary pressure at $z > l$ [Pa]
p_a	the air pressure [Pa]
p_w	the pressure in water [Pa]
Q	dimensionless flux
$\mathbf{n}$	vector of normal [m]
t	time [s]
$\mathbf{V}$	dimensionless velocity
$\mathbf{V_{ev}}$	dimensionless evaporation rate
$\mathbf{V_n}$	dimensionless normal velocity of the phase transition interface
$\mathbf{V_w}$	dimensionless water filtration velocity
$\mathbf{V_{wz}}$	Z-component of the dimensionless water filtration velocity V_w
$\mathbf{v}$	velocity [m s^{-1}]
$\mathbf{v_{ev}}$	evaporation rate [m s^{-1}]
$\mathbf{v_n}$	normal velocity of the phase transition interface [m s^{-1}]
$\mathbf{v_w}$	water filtration velocity [m s^{-1}]
X	dimensionless horizontal coordinate
x	horizontal coordinate [m]
Z	dimensionless vertical coordinate
Z_0	dimensionless Z-coordinate of the characteristic line at $\tau = 0$
z	vertical coordinate [m]
μ_w	water viscosity [Pa s]
ρ_w	water density [kg m^{-3}]
ϕ	porosity [1]
τ	dimensionless time

interface pressure jump was given by the one-dimensional curvature. The finite-time singularity in the solutions of the droplet-breakup problem in the Hele-Shaw cell was observed in [17]. The topological reconfiguration of fluid interfaces was studied in [18] the context of Darcy's law in the case of the Rayleigh-Taylor instability of stratified fluid layers in Hele-Shaw flow. Pinchoff and reconnection in binary fluid flow in a Hele-Shaw cell and the effects of chemical diffusivity, buoyancy, viscous, diffusional and surface tension forces were investigated in [19,20].

In the soils, because of the presence of capillary forces, there is no sharp interface between the regions of saturated water and air [21]. However, in some cases, the presence of an unsaturated region is neglected and a sharp interface is introduced. For example, when the characteristic scale of the problem length is much larger than the size of the unsaturated region [22], or if there is a dynamic equilibrium of the unsaturated zone with the flow of saturated groundwater, then the analysis of the unsaturated zone is not necessary [23]. In these cases, a sharp-interface model is used to simulate the flow with the interface. Similar sharp-interface models are used, for example, to simulate the injection of carbon dioxide into underground reservoirs [25], for simulating groundwater flow phenomena subject to salty water intrusion [24,26] and to investigate stability of salinity profile [27].

In some cases liquid moves under action of capillary pressure only. A mechanism of imbibition is analyzed using a simple piston-like displacement model that has been confirmed by the results of experimental investigations [3,29].

The formulation of the problem proposed in this paper is accurate for flows in fractures or the Hele-Shaw cell, when a two-phase region does not arise and we use sharp-interface approximation for describing flows in a porous medium.

The paper is organized as follows. In the second section, the problem of deformation of a water-saturated region under the influence of capillary pressure acting at the interface is formulated, and a numerical method for solving this problem is briefly described. In the third section, the deformation of an elongated rectangle is considered. In the asymptotic case, an approach similar to the shallow water approximation is used. An analytical solution of the problem is presented, and it is compared with the results of numerical simulation. The deformation of the region at long time scale is studied, which leads to the desintegration of the rectangle into two disconnected parts. In the fourth section, the deformation of the circular region is investigated and characteristic scenarios of the deformation and desintegration of the region under the influence of the capillary pressure gradient are described. Finally, the last section is devoted to the reverse process—merging of two regions; this process is also realized under the influence of capillary pressure that varies at the water–humid air interface.

2. Problem formulation and numerical method

Capillary pressure depends on the physical properties of the solid particles composing the soils or rocks, and also on the properties of liquids and gases that saturate them. Thus, contact angle depends on the surface contamination, the heterogeneity of adsorption and the roughness of the solid surface [1,3,28]. Also, the surface tension coefficient depends on the temperature.

If in the soils or rocks there are two layers with different properties or differ in the structure of the pores, the capillary pressures, between the two components of their saturating, are different. As a rule, there is a transition zone between the layers, characterized by a gradual change in the macroscopic properties. For example, in the soil, a layer of sand is separated from the clay layer by loam or in a certain volume of soil or rock, the zone of partial contamination separates the clean area from completely polluted. The presence of a temperature gradient also creates a capillary pressure gradient due to a change in the surface tension coefficient. Within the framework of the macroscopic approach, the capillary pressure varies smoothly from one value to another in the transition layer.

Consider a porous medium, consisting of two layers A and B of different properties, saturated with humid air that includes a simply connected water-saturated region C (Fig. 1). We assume that there is sharp interface between the regions of saturated water and air [22]. This piston-like model is one of the easiest ways to describe the movement of the boundary of the region C (Fig. 1). This approach can be applied when typical length scale of the problem is sufficiently larger than intermediate zone saturated with the