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Modified likelihood Kalman filter for systems with incomplete, delayed and lost measurements

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ABSTRACT

In this note, a novel Kalman filter is developed in the Bayesian framework for linear dynamical systems whose outputs are measured by faulty sensors and transmitted to the filter through a lossy delaying channel. The main novelty of the proposed method is to modify the likelihood function of the common Kalman filter to cope with incomplete, delayed and lost measurements. The suggested modified likelihood filter can be interpreted as an adaptive Kalman filter, wherein weighting factors are tuned based on the characteristics of the received measurements. Estimation accuracy is assured provided that some conditions on the properties of the sensor and the channel are met. Simulation results are presented to demonstrate the superior performance of the introduced filter compared to some rival ones in the literature.

1. Introduction

Inferring the value of states of dynamical systems from uncertain measurements is called state estimation which is needed in the implementation of feedback control, monitoring and fault diagnosis [1,2]. Kalman filter (KF) as a standard tool for state estimation utilizes recent measurements to achieve a newer estimation. The primary assumption in the conventional KF is that the measurements are immediately accessible at every time instant; however, in many real-world applications such as networked control systems and target tracking via sensor networks, they are acquired by faulty sensors and exposed to random delays and dropouts [3,4].

Based on various models of data latency and dropout, a lot of filtering schemes have been proposed to enhance the estimation accuracy [5]. Concepts of state augmentation and reorganization of observations were employed in the literature to handle delays and dropouts in the design of KF [6]. In [7], an augmented state KF (ASKF) was proposed to solve the out of sequence measurements (OOSMs) problem in the Bayesian framework. In [8], a filtering scheme was proposed considering random delays, data dropouts and missing measurements. Different augmented submodels were defined corresponding to the each situation of the received observations. In [9], an optimal filter dependent on the probabilities of the channel was presented in the minimum variance sense to cope with random measurement delays and losses. In [10], an optimal minimum variance estimator, dependent on packet arriving rate was proposed for systems with finite step correlated noises and packet dropout. In [11], a minimum variance filter was designed based on measurements reorganization idea, considering delays and dropouts of observations. In [12], using projection theorem, an optimal filter was designed for the case that the filter may receive one or multiple measurements at a time or nothing at all. In [13], a new optimal filter was derived based on projection method for linear systems subject to missing measurements with known probabilities.

In this paper, a novel KF is designed in the Bayesian framework for linear discrete-time systems whose outputs are measured by faulty sensors and transmitted via a delaying lossy medium. The key idea of the paper is to modify the likelihood function of the Bayesian filtering method to cope with sensor and communication imperfections. First, the system dynamics is reformulated as an augmented state space model; then, the likelihood function is calculated based on the possible incoming measurements to obtain the recursive equations of the modified likelihood KF (MLKF). The proposed filter which is based on state augmentation notion, like [9] and [8], can be interpreted as a KF with adaptive gains, wherein weighting factors are adjusted based on statistical characteristics of the measurements. Compared to [9] and [8], MLKF has simple structure which leads to low computational burden with better estimation accuracy; moreover, implementation of the suggested filter does not require the exact model of the underlying communication channel. It is worth noting that in general, the filtering performance may fail, because of approximation made in deriving the filter equations. So, conditions on the properties of sensor and the channel are imposed to guarantee the estimation accuracy.

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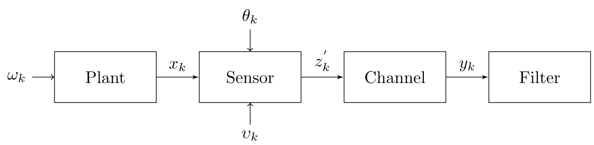

Fig. 1. Schematic diagram of the considered filtering scheme.

The remainder of this paper is organized as follows. Section 2 introduces the models of system and measurements and describes the Bayesian approach to KF design problem. Section 3, presents main results of the paper wherein MLKF relations are derived. In Section 4, the advantages of the suggested MLKF are illustrated. Section 5 concludes the paper.

2. Problem statement and preliminaries

Consider the discrete-time linear stochastic system

$$x_k = A_{k-1}\, x_{k-1} + B_{k-1}\, \omega_{k-1}, \tag{1}$$

$$z_k = C_k\, x_k + D_k\, \upsilon_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector with initial condition x_0 that has a Gaussian distribution, $z_k \in \mathbb{R}^m$ is the ideal output, $\omega_k \in \mathbb{R}^r$ and $\upsilon_k \in \mathbb{R}^p$ are process and measurement noises which are supposed to be white Gaussian with zero mean and covariances $Q_k > 0$ and $R_k > 0$, respectively. A_k, B_k, C_k and D_k are known matrices with appropriate dimensions. It is assumed that the matrix D_k has full row rank; also, x_0 and noises are uncorrelated. In practice, because of sensor aging and temporal failure, the measurement information might be degraded or missed randomly [4]. So, the filter receives real measurements, (3) instead of the ideal ones, (2)

$$z_k' = \theta_k\, C_k\, x_k + D_k\, \upsilon_k, \tag{3}$$

where the random variable $\theta_k = diag\{\alpha_k^1,\ \alpha_k^2,\ \dots,\ \alpha_k^m\}$ describes the faults in the structure of sensors. Independent variables $\alpha_k^i\,(i = 1, 2, \dots, m)$ with the PDF $\rho_k^i(s)(i = 1, 2, \dots, m)$ on the interval [0, 1], are independent of all other noises. It is assumed that the expectation value, $\bar{\theta} = E[\theta_k]$ is available as [4]. As shown in Fig. 1, sensor outputs (3) are transmitted through a communication channel to a filter implemented in a processing center. The possible incomes of the filter are formulated as (4)

$$y_k = \phi,\ \ or\ \ y_k = D_l\, \upsilon_l,\ \ or$$

$$y_k = z_{k-i}', \quad i = 0, 1, \dots, d, \tag{4}$$

where $y_k = \phi$ means that the filter receives nothing (data dropout at time k); i is the delay value with maximum d and $y_k = D_l\, \upsilon_l$ stands for missing measurement, where the filter only receives noise ($\theta_k = 0$). Note that l represents the time that the measurement is missed. In other cases, possibly degraded measurements are received with/without delay ($\theta_k \neq 0$). So, y_k depends on the channel and sensor characteristics. The estimator does not have any information about the exact value of delay, missing and dropout, but only their probabilities.

The aim of this paper is to design a filter in the Bayesian framework to accurately estimate x_k given y^k (all of the received measurements up to time k) without the need to exact mathematical model of the communication link, in contrary to [12]. Before proceeding, the basics of the derivation of KF from the Bayesian viewpoint is briefly recalled from [2]. In the case of ideal measurements, i.e., for systems described by (1) and (2), the filter is designed based on the Bayesian relation

$$p\big(x_k|z^k\big) = \frac{p\big(z_k|x_k\big)}{p\big(z_k|z^{k-1}\big)}\, p\big(x_k|z^{k-1}\big), \tag{5}$$

wherein z^k symbolizes all the measurements up to and including time k and $p\big(x_k|z^k\big)$ is the posterior PDF of state x_k given z^k, $p\big(z_k|x_k\big)$ is the measurement likelihood function, $p\big(x_k|z^{k-1}\big)$ shows the prediction PDF and $p\big(z_k|z^{k-1}\big)$ stands for the normalization factor. By assuming a Gaussian density for the prior PDF of states, namely $p\big(x_{k-1}|z^{k-1}\big)$, KF relations are extracted from (5) by some straightforward manipulations.

However, in the considered problem, incomplete, delayed and lost measurements, y_k are received by the filter instead of z_k; so, (5) is not usable now. As the relation between y_k and x_k is not in the form of (2), the calculation of the posterior PDF of x_k given uncertain measurements up to time k, i.e. y^k is challenging. The complete information of the states of the dynamical system (1) can be described by the joint PDF $p\big(x^k\big) = p\big(x_k,\ x_{k-1},\ \dots,\ x_0\big)$. When the relationship between y^k and x^k is known and $p\big(x^k\big)$ is available, the Bayes theory is used to update the knowledge about these states [2] as the following

$$p\big(x^k|y^k\big) = \frac{p\big(y^k|x^k\big)\, p\big(x^k\big)}{p\big(y^k\big)}. \tag{6}$$

The posterior PDF, $p\big(x^k|y^k\big)$ represents the updated version of the prior knowledge $p\big(x^k\big)$ using the newly available information in the likelihood function, $p\big(y^k|x^k\big)$; while, $p\big(y^k\big)$ is a normalization factor. When the measurements are received sequentially over time, they are used as soon as possible to update the PDF of the states. To do this, (6) is rewritten in a recursive form

$$p\big(x^k|y^k\big) = \frac{p\big(y_k,\ y^{k-1}|x^k\big)\, p\big(x_k,\ x^{k-1}\big)}{p\big(y^k,\ y^{k-1}\big)}. \tag{7}$$

According to the causality principle and the Markov property, (7) can be simplified as

$$p\big(x^k|y^k\big) = \frac{p\big(y_k|x_k,\ x_{k-1},\ \dots,\ x_{k-d}\big)p\big(x_k|x_{k-1}\big)}{p\big(y_k|y^{k-1}\big)} \times\ p\big(x^{k-1}|y^{k-1}\big). \tag{8}$$

Now, (8) is marginalized to determine the posterior PDF of x_k as follows

$$p\big(x_k|y^k\big) = \frac{1}{p\big(y_k|y^{k-1}\big)} \int_{x_{k-d}} \cdots \int_{x_{k-1}} p\big(x_k|x_{k-1}\big) \times\ p\big(y_k|x_k, x_{k-1}, \dots, x_{k-d}\big) \times\ p\big(x_{k-1}, x_{k-2}, \dots, x_{k-d}|y^{k-1}\big) \times\ \mathrm{d}x_{k-1} \cdots \mathrm{d}x_{k-d} \tag{9}$$

However, exact calculation of (9) and consequently, designing the filter is intractable. In the next section, the proposed solution to this dilemma is explained.

3. Design of MLKF

The first step to develop the proposed filter is to define the augmented system

$$X_k = \bar{A}_{k-1}\, X_{k-1} + \bar{B}_{k-1}\, \omega_{k-1}, \tag{10}$$

where $X_k = \big[x_k^T,\ x_{k-1}^T,\ \dots,\ x_{k-d}^T\big]^T$ and

$$\bar{A}_k = \begin{bmatrix} A_k & 0 & 0 & \cdots & 0 \\ I & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & I & 0 \end{bmatrix},\ \bar{B}_k = \begin{bmatrix} B_k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$