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Losses loom larger than gains and reference dependent preferences in Bernoulli's utility function[☆]

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ABSTRACT

Some analysts claim that Bernoulli's utility function is "reference-independent", so it is not able to generate a loss aversion index, and that the theoretical framework of Prospect Theory (PT) is required to achieve those results. This paper examines that claim and finds that the geometry of Bernoulli's original utility function specification either explains or implies key elements of PT: reference dependence and a loss aversion index. Theory and evidence show that the loss aversion index constructed from reference wealth in Bernoulli's utility specification is in the domain of attraction of a stable law. That is, its distribution is a slow varying function with a fat tail that decays like a power law. Additionally, the index can be tested with a modified Fisher z-transform test. Bernoulli's utility function also sheds light on why loss aversion may be over-estimated under PT. In a nutshell, Bernoulli's utility function is alive and well.

"What has been will be again, what has been done will be done again; there is nothing new under the sun." Ecclesiastes 1:9

1. Introduction

A recent survey by Barberis (2013, p. 173) describes Kahneman and Tversky (1979) original version of prospect theory (OPT), and its amendment, cumulative prospect theory (CPT) (Tversky and Kahneman, 1992) thusly. "Prospect theory is still widely viewed as the best available description of how people evaluate risk in experimental settings",[1] while duly noting

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[1] In contrast, Moscati (2016) references Gilboa and Marinacci (2013, p. 232) to note that "it is not clear that a single theory of decision making under uncertainty will replace expected utility theory, and "even if a single paradigm will eventually emerge, it is probably too soon to tell which one it will be."

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that "there are relatively few well-known and broadly accepted applications of prospect theory in economics". This paper compares Bernoulli's (1738) model to the Kahneman and Tversky (1979); Tversky and Kahneman (1992) models of prospect theory to see whether concepts like reference point and loss aversion index are *explained or implied* by Bernoulli's original utility function.

In contrast to Barberis, papers by Birnbaum (2005, 2008); Birnbaum and Navarrete (1998) point out that CPT is unable to explain why certain gambles show systematic violations of stochastic dominance[2]–while certain types of rank dependent models can, e.g., configural weighting and transfer exchange (TAX) of weights by Birnbaum and Chavez (1997). Baltussen et al. (2006) found that CPT was unable to explain decision makers choices in mixed gambles with moderate probabilities where stochastic dominance was also violated. Schmidt and Zank (2008) introduced "third generation" CPT to address, *inter alia*, issues related to the uncertain reference point deficit in CPT. However, Birnbaum's (2018) staunch criticism of CPT remains unabated as he indicates that configural weigting models still outperform third generation CPT.

Meanwhile, theorists like Gilboa and Marinacci (2013) are more circumspect. They believe that no single theory has emerged to replace Expected Utility Theory (EUT). Indeed, experiments conducted by Hey and Orme (1994) found that EUT was upheld as a valid model of decision making for many subjects.[3] List (2004) reviewed several experiments where inexperienced subjects tended to behave according to prospect theory, but more experienced subjects tended to behave in accordance with EUT. Papers by Bruhin et al. (2010); Harrison and Rutström (2008a) also find a mixture of CPT and EUT types in their experiments. Brandstätter et al. (2006) posit that many results attained by the "weighting and summing" feature of EUT and CPT can be obtained by a priority heuristics. However, Glöckner and Betsch (2008) provide evidence in favour of CPT against the priority heuristic.

In a critical review paper, Gal and Rucker (2018) noted that Daniel Kahneman stated that loss aversion is perhaps the most useful contribution of prospect theory to behavioral decision theory (Kahneman, 2002). They argue that "most writings on loss aversion appear to accept the assumption that losses do loom larger than gains and deviations from this are aberrations and violations of the norm that do not challenge the basic principle". However, they find that loss aversion is less robust and universal than assumed, and call for "critical reevaluation of prevailing paradigms" of loss aversion. Higgins and Liberman (2018) and Simonson and Kivetz (2018) were more guarded in their response but by and large concurred with Gal and Rucker (2018) challenge to critically reevaluate loss aversion as a behavioural phenomenon. Ert and Erev (2013) question the efficacy of loss aversion as a behavioural phenomenon and highlighted six experimental design patterns that increase the likelihood of loss aversion in subjects. Yechiam (2018) also provides a succinct review of the literature and notes that proponents of loss aversion tend to over-interpret its results.

Wu and Markle (2008) found that the strong gain loss separability assumption, often used to facilitate estimation of CPT's subutility functions as well as a loss aversion index, was not supported by their experiments. Por and Budescu (2013) established the robustness and persistence of strong gain-loss separability violations across multiple elicitation methods. They speculate that decision makers unobserved reference point may play a role in this violation. Charles-Cadogan (2016) introduced a weakly separable rank dependent utility model with reference dependent loss aversion that militates against the strong gain-loss hypothesis. In this paper we show how Bernoulli's original utility function specification deals with gain-loss separability, and help shed light on why loss aversion may be overestimated.

To the extent that Bernoulli (1738) utility theory laid a foundation for Von Neumann and Morgenstern (1953) axiomatic EUT, it is interesting to know which elements of prospect theory are explained or implied by Bernoulli's utility function. This paper undertakes that task. A key motivation for this approach is the relative simplicity of EUT versus the complexity of CPT. Whereas EUT requires a "regular" utility function, and objective probabilities, CPT involves two key transformations. First, a transformation for outcomes that require two different subutility functions called value functions: one over gains, and the other over losses relative to a reference point. Second, a transformation of objective probabilities to probability weighting functions. In the experimental literature this is often accompanied by choice functions to model the stochastic choice of subjects in an experiment. For instance, Stott (2006, p. 102) examined 256 model variants for implementing CPT based on a combination of different specifications for value functions, probability weighting functions, and choice functions popularized in the literature.

Prospect theory was proposed in response to purported anomalies from experiments in psychology and behavioral economics which led to revisions of Von Neumann and Morgenstern (1953) expected utility theory (EUT) model, and utility theory more generally. However, this paper shows that Bernoulli's (1738) *original utility function specification*, which falls under rubric of EUT, explicitly or implicitly satisfies several innovations attributed to prospect theory's construct: (1) gain loss asymmetry, i.e., "losses loom larger than gains",[4] (2) reference dependent preferences,[5] (3) utility valuation over changes in wealth; and (4) support for a loss aversion index. We also show that under mild assumptions Bernoulli's utility function accommodates a Fisher z-transformation test for the loss aversion index. Moreover, the index follows an α-stable law. These

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[2] If A and B are gambles, and for some outcome $x \in A$ and $x \in B$, and threshold x^*, $\Pr\{x > x^* | A\} > \Pr\{x > x^* | B\}$, then A stochastically dominates B.

[3] Other models in the EUT class like regret theory (Bell, 1982; Loomes and Sugden, 1982), disappointment aversion (Gul, 1991), and the more recent weakly separable rank dependent utility theory (Charles-Cadogan, 2016) explain, *inter alia*, phenomena like preference reversal, Allais paradox, and loss aversion that are in the solution space of prospect theory.

[4] (See e.g., Kahneman and Tversky, 1979, p. 279).

[5] Kőszegi and Rabin (2006, p. 1134) "build on the essential intuitions in Kahneman and Tverskys [1979] prospect theory and subsequent models of reference dependence" but failed to acknowledge reference dependence in Bernoulli's utility function specification.