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Factor substitution and convergence speed in the neoclassical model with elastic labor supply

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HIGHLIGHTS

- I examine the effect of factor substitution on the speed of convergence.
- I consider the Ramsey model with elastic labor supply.
- Negative link if the baseline effective capital is below its steady state.

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ABSTRACT

We study the link between factor substitutability and the speed of convergence in the Ramsey–Cass–Koopmans model with elastic labor supply and normalized CES production. If the baseline value of capital per unit of effective labor is below its steady-state value, an increase in the elasticity of substitution reduces the convergence speed.

1. Introduction

This paper analyzes the effect of the elasticity of substitution on the speed of convergence in the Ramsey–Cass–Koopmans model with elastic labor supply. The theoretical study of the determinants of the speed of convergence is interesting because it determines the importance of transitional dynamics relative to the steady state. This may have important consequences, e.g., for the evaluation of growth-promoting policies. Thus, this issue has been the subject of an active research (e.g., Ortigueira and Santos, 1997; Eicher and Turnovsky, 1999; Chatterjee, 2005; Gómez, 2008; Groth and Wendner, 2014).

de La Grandville (1989) and Klump and de La Grandville (2000) show that the elasticity of substitution is a powerful engine of growth in the Solow model. Specifically, a higher elasticity of substitution generates a higher per capita capital and income in the steady state and along the transition path. Subsequent research has determined the effect of factor substitution on steady-state magnitudes in the Ramsey–Cass–Koopmans (RCK) model (Klump, 2001; Irmen and Klump, 2009; Xue and Yip, 2012; Gómez, 2018), the Diamond overlapping generations model (Miyagiwa and Papageorgiou, 2003; Xue and Yip, 2012), the endogenous growth model with physical and human capital (Gómez, 2015, 2017) and the Barro model (Gómez, 2016).

The steady-state analysis has often been combined with the study of the effect of factor substitution on the speed of convergence. Thus, Klump and Preissler (2000) study this effect in the Solow model, and Klump (2001), Klump and Saam (2008) and Xue and Yip (2012) extend their analysis to the RCK model. Ţhey find that if the baseline level of per capita capital is below its steady-state value –the most realistic case (e.g., Klump et al., 2012) – a higher elasticity of substitution entails a lower convergence speed. Xue and Yip (2012) show that in the Diamond model a higher elasticity of substitution entails a lower convergence speed if the baseline level of per capita capital is above its steady-state value. However, this literature has considered growth models in which labor supply is inelastic. Barro and Sala-i-Martin (2004) study the convergence speed in the RCK model with elastic labor supply, but considering the simplest Cobb–Douglas technology rather than the (normalized) CES technology. Thus, they do not consider the effect

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of factor substitution on the adjustment speed. Our purpose is to fill this gap. We find that if the baseline value of capital per unit of effective labor is below its steady-state value, an increase in the elasticity of substitution reduces the speed of convergence. Thus, our result extends and confirms those previously obtained for the Solow model and the RCK model with inelastic labor supply.

This paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effect of the elasticity of substitution on the speed of convergence. Section 4 concludes.

2. The neoclassical model with elastic labor supply

We consider a closed economy populated by a unit measure of identical, infinitely-lived households. The household's size N grows at the constant rate n and, for simplicity, the initial size $N(0)$ is normalized to unity. Each member of the household is endowed with one unit of time which can be allocated to work, u, or leisure, $1-u$.

Aggregate output is produced using aggregate capital NK and labor $L=uN$ by means of the CES technology

$$NY = F(NK, L) = B[\alpha(NK)^{\psi} + (1-\alpha)L^{\psi}]^{1/\psi},$$
$$B>0,\ \ 0<\alpha<1,\ \ \psi<1,$$

where Y is per capita output and K is per capita capital. Here B is the productivity parameter, α is the distribution parameter and $\sigma = 1/(1-\psi)$ is the elasticity of substitution. Let $y = Y/u$ denote the effective production; i.e., production per unit of labor, and let $k = K/u$ denote the effective capital. The production function in intensive form can be written as

$$y = f(k) = F(k,1) = B[\alpha k^{\psi} + (1-\alpha)]^{1/\psi},$$

and

$$\frac{\partial F}{\partial K}(K,u) = f'(k) = \alpha B^{\psi}[f(k)/k]^{1-\psi}, \tag{1}$$

$$\frac{\partial F}{\partial u}(K,u) = f(k) - kf'(k). \tag{2}$$

The representative household derives utility from (per capita) consumption C and disutility from work time u in accordance with the intertemporal utility function

$$U = \int_0^{\infty}\left[\ln C - v\frac{u^{1+\eta}}{1+\eta}\right]e^{-(\rho-n)t}dt,$$
$$\eta>0,\ \ v>0,\ \ \rho>n>0. \tag{3}$$

Here, ρ is the rate of time preference and η is the inverse of the Frisch elasticity. The household maximizes (3) subject to the budget constraint which, in per capita terms, is

$$\dot{K} = F(K,u) - C - (n+\delta)K, \tag{4}$$

where δ is the depreciation rate. The first-order conditions of this problem are

$$\frac{1}{C} = \lambda, \tag{5}$$

$$vu^{\eta} = \lambda\frac{\partial F}{\partial u}(K,u), \tag{6}$$

$$\dot{\lambda} = \left[\rho + \delta - \frac{\partial F}{\partial K}(K,u)\right]\lambda, \tag{7}$$

where λ is the shadow value of capital, together with the standard transversality condition.

Using (5), (6) and (2) per capita consumption can be expressed as a function of k and u as

$$C = C(k,u) = \frac{f(k)-kf'(k)}{vu^{\eta}}. \tag{8}$$

Log-differentiating (5), using (7) and (1), we get the evolution of per capita consumption as

$$\frac{\dot{C}}{C} = f'(k) - \rho - \delta. \tag{9}$$

The resources' constraint in per capita terms can be expressed as

$$\dot{K} = u[f(k)-(n+\delta)k] - C = u[f(k)-(n+\delta)k] - \frac{f(k)-kf'(k)}{vu^{\eta}}. \tag{10}$$

Log-differentiating (8) with respect to time, after simplification, we get that

$$\frac{\dot{C}}{C} + \eta\frac{\dot{u}}{u} = -\frac{kf''(k)}{f(k)-kf'(k)}\dot{k}. \tag{11}$$

We also have that

$$\dot{k} = k\left[\frac{\dot{K}}{K} - \frac{\dot{u}}{u}\right] = \frac{\dot{K}}{u} - k\frac{\dot{u}}{u}. \tag{12}$$

Solving the system (11)–(12) we get that

$$\dot{k} = \frac{k[f(k)-kf'(k)]}{\eta[f(k)-kf'(k)] - k^2 f''(k)}\left[\frac{\dot{C}}{C} + \eta\frac{\dot{K}}{uk}\right], \tag{13}$$

$$\dot{u} = -\frac{u}{\eta}\left[\frac{\dot{C}}{C} + \frac{kf''(k)}{f(k)-kf'(k)}\dot{k}\right]. \tag{14}$$

The system (13) and (14) describes the dynamics of the economy in terms of k and u, where (8), (9) and (10) are used to substitute for $C = C(k,u)$, $\dot{C}$ and $\dot{K}$, respectively.

In the steady state effective capital and labor supply are constant; i.e., $\dot{k} = \dot{u} = 0$. If there exists a stationary solution, equating (14) to zero it must be that $\dot{C} = 0$. Hence, equating (13) to zero it must be that $\dot{K} = 0$, so we have that

$$f'(\bar{k}) = \rho + \delta, \tag{15}$$

$$\bar{u} = \left[\frac{f(\bar{k}) - \bar{k}f'(\bar{k})}{v(f(\bar{k}) - (n+\delta)\bar{k})}\right]^{1/(1+\eta)}. \tag{16}$$

Taking into account that

$$\lim_{k\to 0} f'(k) = \begin{cases} +\infty, & \text{if } \psi \geq 0,\\ B\alpha^{1/\psi}, & \text{if } \psi < 0,\end{cases}$$

and

$$\lim_{k\to +\infty} f'(k) = \begin{cases} B\alpha^{1/\psi}, & \text{if } \psi > 0,\\ 0, & \text{if } \psi \leq 0,\end{cases}$$

together with $f''(k) < 0$, the condition $\lim_{k\to 0} f'(k) > \rho + \delta > \lim_{k\to +\infty} f'(k)$ is necessary and sufficient for the existence of a unique steady state. Thus, we shall make the following assumption:

Assumption 1. Parameter values are so that $\rho + \delta > B\alpha^{1/\psi}$ if $\psi > 0$ and $\rho + \delta < B\alpha^{1/\psi}$ if $\psi < 0$.

3. Elasticity of substitution and convergence speed

de La Grandville (1989) and Klump and de La Grandville (2000) have stressed the importance of normalizing the underlying CES production function "to gauge the sensitivity of results (steady-state or dynamic) to variations in the substitution elasticity" (Klump et al., 2012, p. 792). Therefore, we shall first briefly describe the normalization procedure.