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A new integrated AR-IDEA model to find the best DMU in the presence of both weight restrictions and imprecise data

Bohlool Ebrahimi[a,b,*], Masoud Khalili[c]

[a] Satellite Research Institute, Iranian Space Research Center, Tehran, Iran
[b] Industrial Engineering, Technology Development Institute, ACECR, Sharif university branch, Tehran, Iran
[c] Payame Noor University, Tehran, Iran

A R T I C L E I N F O

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A B S T R A C T

Data envelopment analysis (DEA) is a mathematical programming approach to efficiency measurement in many applications. In some real-life applications, three situations are established. First, the input and output data are imprecise. Second, it is necessary to use the weight restrictions in order to consider the management view. Third, the decision maker needs to find the best decision making units (DMUs). In the traditional DEA model, it is supposed that the input and output data are precise and the weights are free. This paper proposes a new mixed integer assurance-region imprecise DEA (AR-IDEA) model to find the best DMU by solving only one model. Also, a new algorithm is developed to find and rank the other efficient DMUs by using the proposed model. Indeed, the proposed approach can be used to find and rank the best DMUs in real-life applications. A numerical example of the supplier selection problem is provided to show the usefulness and effectiveness of the proposed approach.

1. Introduction

Data envelopment analysis (DEA) developed by Charnes, Cooper, and Rhodes (1978) is a mathematical programming approach for performance evaluation of a set of DMUs that use several inputs to produce several outputs. Suppose that we have N DMUs for performance evaluation. In this case, DEA solves N mathematical models to determine the efficient and inefficient DMUs. It should be noted that more mathematical models are needed to be solved to find the best DMU among the efficient DMUs. In the recent decade, some researchers have developed the different mathematical models to find the best DMU by solving only a mixed integer DEA model. These models are reviewed in the next section.

The traditional DEA models make an assumption that the input and output data have exact values on a ratio scale. However, in many real-life applications, the data may be imprecise such as bounded, ordinal, ratio bound, and so on. For the first time, Cooper, Park, and Yu (1999) addressed the problem of imprecise data in DEA. In the literature of DEA, different approaches are developed to handle the imprecise data in the DEA (see e.g. Ebrahimi, Tavana, & Rahmani, 2016).

Furthermore in the DEA model, the DMUs are free to choose the weights in order to maximize their relative efficiencies. Consequently, some input and output weights may take very small or large values in the performance evaluation process. Moreover, these weights are often in contradiction with the management views about the importance of criteria (inputs and outputs) or additional available information. To overcome these shortcomings and problems, different types of weight restrictions are proposed that are briefly studied in the next section.

Overall as explained above, for efficiency measure in the real-life applications with the DEA model, we need to consider the following conditions:

- The decision maker (DM) needs to find the best DMUs to make a decision.
- The input and output data of DMUs include imprecise data.
- It is necessary to use the weight restrictions to prevent unusual weights and also to consider the management views.

The main goal of this paper is to develop a new DEA model that enables the DM to determine the best DMUs in the presence of both imprecise data and weight restrictions. Consequently, instead of solving at least one optimization problem for each DMU, an integrated model can be solved to find the best DMU. Moreover, we propose a new algorithm to find and rank other efficient DMUs. The remainder of this paper is organized as follows: in Section 2, the literature review is presented. In Section 3, the new developed approach to find and rank the efficient DMUs in the presence of both imprecise data and weight restrictions is presented. Numerical example and conclusions are given

* Corresponding author at: Satellite Research Institute, Iranian Space Research Center, Tehran, Iran.
E-mail addresses: b_ebrahimi@jdsharif.ac.ir, b.ebrahimi@aut.ac.ir (B. Ebrahimi).

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in Sections 4 and 5, respectively.

2. Literature review

In this section, a brief review of DEA model in the presence of weight restrictions and imprecise data is given. Then, the existing integrated DEA models to find the best DMUs by solving only one mathematical model are studied. The linear CCR (Charnes et al., 1978) model for evaluating the DMU_p is as follows:

$$\begin{aligned} &\max \sum_{r=1}^{m} u_r y_{rp} \\ &s.t. \\ &\quad \sum_{i=1}^{n} v_i x_{ip} = 1 \\ &\quad \sum_{r=1}^{m} u_r y_{rj} - \sum_{i=1}^{n} v_i x_{ij} \leqslant 0, \quad j = 1,\dots,k \\ &\quad u_r,\, v_i \geqslant 0 \end{aligned} \tag{1}$$

where y_{rj} and x_{ij} are the output and input vectors of DMU_j respectivelyand $u = (u_1, u_2, \dots, u_m)$ & $v = (v_1, v_2, \dots, v_n)$ are the vectors of output and input weights, respectively. In the model (1), it is supposed that all input and output data are specific numerical values. In some real world applications, however, some input and output data may be measurable only on imprecise data. This situation occurs when the DMUs include judgment, forecasting and missing data or ordinal preference information. Imprecise data or inexact information can be stated in ordinal, difference order, multiplied order, interval (bounded) and Ratio bound data or fuzzy data. Thus, it is worthy to study more about the evaluation of DMUs in imprecise environments. There are a lot of papers considering the model (1) in the presence of fuzzy data that can be categorized into four groups: the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach (Hatami-Marbini, Emrouznejad, & Agrell, 2011). In this paper, we focus on different types of imprecise data containing interval (bounded), ordinal, ratio bound, difference order and multiplied order data. The interested readers can refer to the articles of Hatami-Marbini et al. (2011), and Muren and Wei (2014) for more study about the fuzzy DEA models.

For the first time, Cooper et al. (1999) studied the bounded and weak ordinal data in DEA and named the new model as imprecise DEA (IDEA), which was a nonlinear model. Until now, different approaches are developed to calculate the relative efficiency scores with the model (1) in the presence of imprecise data. The existing approaches in this area can be divided in three groups. The first group ranks the DMUs based on only the upper bound efficiencies (e.g. Cooper et al. (1999, 2001); Zhu (2003, 2004)). The second group used the both of the lower bound and upper bound efficiencies to rank the DMUs (e.g. Despotis and Smirlis, 2002; Wang, Greatbanks, & Yang, 2005; Kao, 2006; Park, 2007; Hatami-Marbini, Emrouznejad, & Tavana, 2014). The third group used the efficiency distribution and the expected efficiencies to rank the DMUs (e.g. Kao and Liu, 2009; Ebrahimi et al. (2016, 2017)). The proposed approaches in the third group can produce better results in comparing with the first and second groups, but they have a high volume of calculations in large scale problems.

In addition, different types of weight restriction are developed in the model (1) to prevent unusual weights and also to consider the management views. The most popular type of weight restrictions is linear constraints that are presented in the Table 1 (Khalili, Camanho, Portela, & Alirezaee, 2010).

In the presence of ARI, the model (1) is always feasible and can calculate the relative efficiencies correctly. However, some problems

Table 1
Linear weight restrictions.

Linear weight restrictions	Mathematical Formula
Assurance region, type 1 (ARI)	$\alpha_i \leqslant v_i/v_{i+1} \leqslant \beta_i, \quad \lambda_r \leqslant u_r/u_{r+1} \leqslant \theta_r$
Assurance region, type 2 (ARII)	$\gamma_i v_i \geqslant u_r$
Absolute weight restrictions	$\delta_i \leqslant v_i \leqslant \tau_i, \quad \rho_r \leqslant u_r \leqslant \eta_r$

Where θ_r, α_i, β_i, γ_i, δ_i, τ_i, ρ_r, η_r, λ_r are scalar.

such as infeasibility and underestimation of the efficiencies may be occurred by using the absolute weight restrictions or ARII (Khalili et al., 2010). More recently, Podinovski and Bouzdine-Chameeva (2013, 2015) extracted some other problems of using the linear weight restrictions, such as zero or negative efficiency scores. It should be noted that all of the mentioned problems occur in the presence of ARII or the absolute weight restrictions. In other words, no problem occurs in the performance evaluation process by using the ARI in the model (1). This may be the main reason of the widespread use of this type of weight restriction in real-world applications of DEA (see, e.g. Thompson, Lee, & Thrall, 1992 for 45 gas producers; Schaffnit, Rosen, & Paradi, 1997 for branches of a large Canadian bank; Olesen and Petersen, 2002 for measuring the efficiency of hospitals and Farzipour Saen, 2008 for supplier selection problem). So, we will use the ARI in our developed approach.

2.1. Finding the best DMUs

By using the model (1), we need to solve at least one linear programming (LP) for each DMU to find the best DMU. Recently, as explained in the previous section, some new models are developed to find the best DMU by solving only one model studied in this section. Amin and Toloo (2007) proposed the following model (2) for finding the most efficient DMU.

$$\begin{aligned} &M^* = Min \quad M \\ &s.t. \quad M - d_j \geqslant 0 \quad j = 1,\dots,k \\ &\quad \sum_{i=1}^{n} v_i x_{ij} \leqslant 1 \quad j = 1,\dots,k \\ &\quad \sum_{r=1}^{m} u_r y_{rj} - \sum_{i=1}^{n} v_i x_{ij} + d_j - \beta_j = 0 \quad j = 1,\dots,k \\ &\quad \sum_{j=1}^{k} d_j = k-1 \\ &\quad 0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0, 1\} \quad j = 1,\dots,k \\ &\quad u_r,\, v_i \geqslant \varepsilon^* \quad \forall\, r, i \end{aligned} \tag{2}$$

where d_j is a binary variable representing the deviation variable of DMU_j from the efficiency. β_j is a continuous variable, and M is the maximum inefficiency which should be minimized. ε^* is the optimal non-Archimedean epsilon.

It should be emphasized that finding an appropriate value for ε^*, is a challenging issue. In other words, selecting a very big value for ε^* makes the model infeasible and a very small value of ε^* lets the weights get zero values. Charnes, Rousseau, & Semple (1993) showed that an inappropriate value for ε^* may lead to infeasibility or unboundedness in DEA models. For more details about the role of epsilon in DEA models we refer the reader to Salahi & Toloo (2017). Therefore, Amin and Toloo (2007) proposed a model to determine a suitable value for ε^*.

Toloo & Nalchigar (2011) directly added the imprecise data into the model (2) to obtain the following nonlinear model (3).