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Planning of step-stress accelerated degradation test based on non-stationary gamma process with random effects

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ABSTRACT

This paper discusses the design problem of the step-stress accelerated degradation test (SSADT) based on the non-stationary gamma process with random effects. The cumulative exposure (CE) model is used to link the degradation paths of the SSADT under different stress levels. The expectation maximization (EM) algorithm is applied to estimate the model parameters. The purpose is to design an optimal experiment plan by minimizing the asymptotic variance of the estimated reliability of the product at a predesigned mission time of interest. Under the budget and boundary constraints, the design variables such as sample size, the measurement frequency at each stress level, and the number of measurements at each stress level are obtained. In the end, an example about the light emitting diode (LED) chip is used to illustrate the proposed model.

1. Introduction

The research on degradation test has gained much attention in recent years since the quality of products is getting higher and higher, and the lifetime data are more difficult to obtain. In degradation tests, the quality characteristics (QCs) are measured to assess the reliability of the products (Hao & Yang, 2018). Under the normal test conditions, the degradation data are also difficult to obtain in finite time for some highly reliable products. Therefore, an accelerated degradation test (ADT) conducted at severe environmental conditions has been developed to shorten the time for collecting data (Meeker, Escobar, & Lu, 1998). The ADT includes constant-stress accelerated degradation test (CSADT), step-stress accelerated degradation test (SSADT) or progressive-stress accelerated degradation test (PSADT) (Meeker & Escobar, 1998). Among them, the CSADT and SSADT are widely applied, since they are easy to monitor. Besides, the SSADT has another merit that only a small number of samples are required.

In reality, many products degrade stochastically with time. Therefore, the ADT based on stochastic process, such as the Wiener process (Jin & Matthews, 2014; Hu, Lee, & Tang, 2015), gamma process (Pan & Sun, 2014; Tsai, Sung, Lio, Chang, & Lu, 2016) or inverse Gaussian (IG) process (Wang & Xu, 2010; Ye & Chen, 2014), has been extensively developed. Furthermore, the degeneration paths of some products are non-linear increasing, so the non-stationary gamma process can be used to model the degradation process. About the non-stationary gamma process, it was initially defined by Berman (1981). Bandyopadhyay and Sen (2005) discussed a special class of non-stationary gamma process called modulated power law process. Wang (2008) studied the semiparametric pseudo-likelihood inference for non-stationary gamma process with random effects for degradation data. Wang (2009) considered the nonparametric estimation of the shape function of gamma process model. Verma, Srividya, and Rana (2011) estimated the mean-time-to-failure (MTTF) of non-stationary gamma process. The approximation formula could be easily applied to most of the mechanical components. Wang, Balakrishnan, Guo, and Jiang (2015) discussed the remaining useful life (RUL) estimation problem for the bivariate non-stationary gamma process using the copula function. Guida, Postiglione, and Pulcini (2018) provided a Bayesian approach to conduct the statistical inference for the non-stationary gamma degradation processes.

Besides the statistical inference, the experimental design problem is also an important issue for degradation test (Yu, 2007). Lim and Yum (2011) studied the optimal design issue of the CSADT based on the Wiener process. The test stress levels and the proportion of units allocated to each stress level were determined by minimizing the estimated q-quantile of the product's lifetime distribution. Tsai, Tseng, and Balakrishnan (2012) considered the problem of optimal design for degradation tests based on gamma process with random effects. Lim (2015) addressed the optimal CSADT design problem for the gamma process under the constraint of total cost. Tsai et al. (2016) discussed

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the optimal design of two-variable ADT plan for gamma degradation processes. Tseng and Lee (2016) developed the optimal allocation rule for the ADT based on an exponential-dispersion (ED) degradation model. The optimum allocation rules for the ADT with two stress levels and three stress levels under the V-optimality criterion were derived analytically. Li, Hu, Zio, and Kang (2017) presented a Bayesian method to obtain the optimal ADT plan based on the IG process considering three objective functions: relative entropy, quadratic loss function, and Bayesian D-optimality. Wang, Zhao, Ma, and Wang (2017a) proposed the M-optimality criterion for obtaining the optimal CSADT plan based on IG process. Duan and Wang (2018a) addressed the optimal design problems for CSADT based on stationary gamma processes with fixed effect and random effect.

The SSADT is an important approach to obtain the degradation data of products. The proportional degradation rate (PDR) model and cumulative exposure (CE) model are two commonly used model to link the degradation paths of the SSADT with stress levels. The PDR model assumes that the average degradation rate is proportional to an exponential function of the stress level. Duan and Wang (2018b) derived the optimal SSADT plans for the IG process based on the PDR model. Zhao, Pan, and Xie (2018) discussed the optimal SSADT design problem with various optimality criteria for the Wiener process from a Bayesian perspective. Li, Hu, Zhou, Li, and Kang (2018) proposed a multi-objective Pareto-optimal approach to obtain SSADT plan for the IG process from a Bayesian perspective. The CE model assumes that the degradation path of the product depends only on the degradation magnitude already accumulated and the current stress level, and has nothing to do with the way of accumulation. The design of the SSADT based on the CE model has been discussed by some scholars. Liao and Tseng (2006) designed an optimal SSADT plan based on the Wiener process. Under the budget constraint, the optimal settings were obtained by minimizing the asymptotic variance of the estimated q-quantile of the product's lifetime distribution. Tseng, Balakrishnan, and Tsai (2009) discussed the optimal SSADT design for the stationary gamma degradation process. The optimal settings were derived by minimizing the asymptotic variance of the estimated MTTF of the products under the budget constraint. Wang, Wang, and Duan (2016) presented the optimal SSADT plan based on the IG process. Amini, Shemehsavar, and Pan (2016) discussed the optimal design problem for the SSADT with random discrete stress elevating times based on stationary gamma degradation process. Wang, Chen, and Tan (2017b) derived an optimal SSADT plan with considering multiple stresses and multiple degradation measures based on Wiener process. Fan and Chen (2017) developed a Bayesian approach to gain the optimal SSADT plan based on stationary gamma process.

As an important degradation process, the statistical inference problem of the non-stationary gamma process has been intensively studied. However, the SSADT experiment design problem has not been discussed. The aim of this research is to find the optimal SSADT plan for products with non-stationary gamma degradation process. Random effects are introduced to describe the heterogeneity between different products. In analysis, the CE model is applied to link the degradation paths of the SSADT at different stress levels. Under the constraint of total experiment cost and the limitation of the upper and lower bounds of the design variables, the optimal settings including sample size, the measurement frequency at each stress level and the number of measurements at each stress level are obtained by minimizing the asymptotic variance of the estimated reliability of the product at a predesigned mission time of interest. In existing works, the measurement frequencies and the operation cost per unit time are usually set to the same value under different stress levels. In this paper, the measurement frequencies and the operation cost per unit time under different stress levels are different, which is more reasonable. For the random effect model, the elements of the Fisher information matrix are always calculated by using the Monte Carlo (MC) method in existing literatures, which results in the low calculation efficiency. In this paper, we prove that the Fisher information matrix of the random effect model can be calculated theoretically, and give a relatively broad lemma to solve the similar computational problem. Finally, an example about light emitting diode (LED) chip is used to validate the proposed model and method.

The remainder of this paper is organized as following. A step-stress ADT model based on non-stationary gamma process with random effects is described in Section 2. Section 3 addresses the parameter estimation problem for the proposed model, and the Fisher information matrix is calculated. The optimization problem with constraints and the algorithm for solving it are also presented in this section. In Section 4, an example about LED chip is used to illustrate the proposed model and method. Finally, some concluding remarks are made in Section 5.

2. A stochastic SSADT model

2.1. Non-stationary gamma degradation process with random effects

The non-stationary gamma process $\{G(t), t \geq 0\}$ with shape function $\rho(t) > 0$ and rate parameter $\beta > 0$ is a stochastic process with following properties (van Noortwijk, 2009):

(i) $G(0) = 0$ with probability one;
(ii) $G(t)$ has independent increments, that is, $G(\tau_2)-G(\tau_1)$ and $G(t_2)-G(t_1)$ are independent for $\forall\ \tau_2 > \tau_1 \geqslant t_2 > t_1$;
(iii) The increment of $G(t)$ follows gamma distribution, i.e., $G(\tau)-G(t) \sim Ga(\beta, \rho(\tau)-\rho(t))$ for $\forall\ \tau > t \geq 0$.

Based on properties (i) and (iii), one can obtain that $G(t) \sim Ga(\beta, \rho(t))$ with mean $\rho(t)/\beta$, variance $\rho(t)/\beta^2$, and probability density function (PDF)

$$f(x|\rho(t), \beta) = \frac{\beta^{\rho(t)}}{\Gamma(\rho(t))} x^{\rho(t)-1} e^{-\beta x}, x > 0, \tag{1}$$

where $\Gamma(\rho(t)) = \int_0^\infty x^{\rho(t)-1} e^{-x} dx$.

To describe the heterogeneity among products, we assume that the rate parameter β follows gamma distribution with parameters δ and γ, i.e. $\beta \sim Ga(\gamma, \delta)$. Eq. (1) gives the PDF of $G(t)$ conditional on β. The unconditional PDF of $G(t)$ is

$$\begin{aligned} f(x|\rho(t), \delta, \gamma) &= \int_0^\infty f(x|\rho(t), \beta)\cdot f(\beta|\delta, \gamma) d\beta \\ &= \frac{x^{\rho(t)-1}\gamma^{\delta}\Gamma(\rho(t) + \delta)}{\Gamma(\rho(t))\Gamma(\delta)(\gamma + x)^{\rho(t)+\delta}}, x > 0. \end{aligned} \tag{2}$$

Theoretical analysis shows that $\delta G(t)/\gamma\rho(t)$ follows the F-distribution with parameters $2\rho(t)$ and 2δ, i.e., $\delta G(t)/\gamma\rho(t) \sim F(2\rho(t), 2\delta)$. For convenience, denote the cumulative distribution function (CDF) of $F(2\rho(t), 2\delta)$ as $F_{2\rho(t),2\delta}(\cdot)$ (Tsai et al., 2012).

Let $\{G(t), t \geq 0\}$ be the non-stationary gamma degradation process of a product with shape function $\rho(t)$ and rate parameter $\beta \sim Ga(\gamma, \delta)$, then the first passage time T of this product is defined as the time when the degradation path crosses a pre-fixed threshold ϖ firstly, i.e., $T = \inf\{t|G(t) > \varpi\}$. The CDF of the lifetime T can be obtained as

$$F_T(t) = P(G(t) > \varpi) = P\left(\frac{\delta G(t)}{\gamma\rho(t)} > \frac{\delta\varpi}{\gamma\rho(t)}\right) = 1-F_{2\rho(t),2\delta}\left(\frac{\delta\varpi}{\gamma\rho(t)}\right). \tag{3}$$

Thus the reliability function is

$$R(t) = F_{2\rho(t),2\delta}\left(\frac{\delta\varpi}{\gamma\rho(t)}\right) = \frac{B_{\frac{\varpi}{\varpi+\gamma}}(\rho(t), \delta)}{B(\rho(t), \delta)} = I_{\frac{\varpi}{\varpi+\gamma}}(\rho(t), \delta), \tag{4}$$

where $B_x(a, b) = \int_0^x t^{a-1}(1-t)^{b-1}dt$ is the incomplete beta function; $B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$ is the beta function; and $I_x(a, b) = B_x(a, b)/B(a, b)$ is the regularized incomplete beta function.