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Phase field modeling of crack propagation under combined shear and tensile loading with hybrid formulation

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ABSTRACT

The crack phase field model has been well established and validated for a variety of complex crack propagation patterns within a homogeneous medium under either tensile or shear loading. However, relatively less attention has been paid to crack propagation under combined tensile and shear loading or crack propagation within composite materials made of two constituents with very different elastic moduli. In this work, we compare crack propagation under such circumstances modelled by two representative formulations, anisotropic and hybrid formulations, which have distinct stiffness degradation schemes upon crack propagation. We demonstrate that the hybrid formulation is more adequate for modeling crack propagation problems under combined loading because the residual stiffness of the damaged zone in the anisotropic formulation may lead to spurious crack growth and altered load–displacement response.

1. Introduction

The initiation and propagation of cracks is one of the main failure mechanisms of engineering materials. Hence, numerous studies have been conducted to develop methods to accurately predict crack initiation and propagation under various mechanical loading conditions to prevent the catastrophic failure of engineering materials and systems. The theoretical basis to predict crack evolution was first introduced by Griffith [1] and Irwin [2]. They addressed the difficulty of dealing with a singular stress field at the crack tip by introducing the concept of the strain energy release, which showed good matching in numerous experiments on the initiation of pre-existing crack growth. However, crack nucleation, curvilinear crack paths, crack branching, or coalescence cannot be well accounted for.

In recent years, there has been an increasing interest in variational approaches to brittle fracture, which is referred to as the crack phase field model [3–7]. The phase field model approximates sharp crack discontinuity with a continuous scalar parameter denoted by the crack phase field. It has been shown that the solution of an approximated crack surface, described by a smooth function, converges to the solutions of sharp crack in the limit of regularization parameter equal to zero [8–13]. The phase field approach has attracted significant attention as a powerful tool to simulate complex crack evolution, including curvilinear crack paths, crack branching, or coalescence. Moreover, the phase field model framework has been extended beyond the linear elastic fracture regime to a wide range of fracture problems, such as large strain problems [14,15], cohesive fractures [16], ductile fractures [17], multi-physics [18–23], pore microstructures [24], and dynamic effects [25,26].

Although extensive phase field modeling studies have been carried out on the failure of homogeneous media, relatively less attention has been paid to the failure of composite materials, which inherently involves complex curvilinear crack propagation paths. In addition to accurately predicting curvilinear crack paths through composites, it is also crucial to obtain the entire stress–strain curve until complete failure to evaluate the toughness modulus of composites [27–29]. However, phase field methodologies [18,19,30,31] based on the early formulation by Miehe [6,7] selectively degrade the stiffness along the direction of the maximum tensile strain upon crack growth. Hence, the cracked region unphysically sustains any subsequent loading with different maximum tensile direction. For example, a straight crack grown under tensile loading would withstand subsequent shear loading, as will be shown later. When the Young's moduli of two constituents in a composite are similar to each other, such behavior does not play a critical role in determining crack paths or predicting stress–strain curves because the crack paths are similar to those of homogeneous materials [31]. However, composites involving two constituents with highly different elastic moduli, such as natural or nature-inspired

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composites, fail due to the propagation of strongly curved cracks that are subject to highly varying combinations of tension and shear loading along the crack paths. In such circumstances, crack paths as well as the stress–strain curves are significantly affected by the aforementioned unphysical load bearing capacity of the cracked regions. For example, composite samples can sustain tensile loading although wavy cracks propagate through the entire sample dimension, as will be shown in this paper.

This paper compares the performance of two different formulations, the early anisotropic formulation by Miehe [6,7] and a recent hybrid formulation [32] for modeling crack propagation in homogeneous materials under a sequence of different loading modes and for simulating strongly curved crack propagation in composite materials. The hybrid formulation, which was originally developed to reduce the computational cost [32], it is demonstrated to not suffer from the aforementioned unphysical load bearing capacity for the case studies considered here.

The remainder of this paper is organized as follows. In Section 2, we review fundamental equations of the phase field approach to quasi-static brittle fracture and briefly introduce the numerical implementation scheme within the commercial software ABAQUS. For ease of comparison with previous studies, we have adopted the notations of Miehe et al. [6,7]. Section 3 provides few modeling examples on crack propagation in homogeneous and heterogeneous media, which highlights the advantage of the hybrid formulation. In Section 4, we summarize the paper and discuss directions for future research.

2. Methods

In this section, we briefly review diffusive crack representation in the phase field framework, and we introduce three different formulations, namely, isotropic, anisotropic, and hybrid schemes, that are classified according to the strain energy split and stiffness degradation scheme upon crack propagation. We then explain the numerical implementation in the commercial finite element software ABAQUS.

2.1. Diffusive crack topology described by crack phase field

Consider a domain $\Omega \subset \mathbb{R}^D$ and its boundary $\partial\Omega$describing a cracked material in D dimensional space (see Fig. 1). Let Γ be a $D-1$ dimensional surface inside of domain Ω. Here, Γ represents the crack surface within the material. As depicted in Fig. 1a, the topology of a sharp crack can be described by the phase field scalar parameter $d(\boldsymbol{x}) \in [0, 1]$ with

$$d(\boldsymbol{x}) = \begin{cases} 1, & on\ \Gamma \\ 0, & otherwise \end{cases}, \tag{1}$$

which represents the fully broken state of the material for $d = 1$ and the unbroken state of the material for $d = 0$ at a given point $\boldsymbol{x}$. In the regularized framework shown in Fig. 1b, the crack topology is approximated by scalar parameter $d(\boldsymbol{x})$ having a unit value on the crack surface Γ and fading away from that surface. The value of the phase field $d(\boldsymbol{x})$ can be determined by solving the following differential equation:

$$\begin{cases} d - l^2\nabla^2 d = 0, & in\ \Omega \\ d(\boldsymbol{x}) = 1, & on\ \Gamma \\ \nabla d(\boldsymbol{x})\cdot\boldsymbol{n} = 0, & on\ \partial\Omega \end{cases}, \tag{2}$$

where $\nabla^2 d$ is the Laplacian of the phase field, $\boldsymbol{n}$ is the outward normal on $\partial\Omega$, and l is the regularization parameter that determines the width of the regularized or diffusive crack topology. To elaborate the concept of the regularization parameter l, a one-dimensional example of a diffusive crack for various values of l is shown in Fig. 2b. In the limit of $l \to 0$, Fig. 2b shows that the diffusive crack topology converges to the ideal sharp crack. Similarly, in two-dimensional and three-dimensional cases, the diffusive crack topology also converges to a sharp crack for vanishing value of l. Diffusive crack topology $\Gamma_l(d)$ can be expressed as

$$\Gamma_l(d) = \int_\Omega \gamma(d, \nabla d)dV, \tag{3}$$

where $\gamma(d, \nabla d)$ is the crack surface density function per unit volume of the material, denoted as

$$\gamma(d, \nabla d) = \frac{1}{2l}d^2 + \frac{l}{2}|\nabla d|^2. \tag{4}$$

In terms of $\gamma(d, \nabla d)$ and the critical energy release rate g_C, we can approximate the surface energy $W(d)$ by volume integral as

$$W(d) = \int_\Gamma g_C dA \approx \int_\Omega g_C\gamma(d, \nabla d)dV. \tag{5}$$

2.2. Strain energy and stiffness degradation of fracturing material

When the strain energy stored at a point of the material exceeds the energy required to open a crack surface, fracture starts and it is accompanied by both strain energy and stiffness degradation. In other words, the crack phase field $d(\boldsymbol{x})$ is driven by the strain energy of the material, and the completely fractured region with $d(\boldsymbol{x}) = 1$ no longer sustains the mechanical loading. To couple the crack phase field $d(\boldsymbol{x})$ with displacement field $\boldsymbol{u}(\boldsymbol{x})$, we define the strain energy of a material $E(\boldsymbol{u}, d)$ as

$$E(\boldsymbol{u}, d) = \int_\Omega \psi(\boldsymbol{\varepsilon}(\boldsymbol{u}), d)dV, \tag{6}$$

where $\boldsymbol{\varepsilon}(\boldsymbol{u})$ is the strain tensor, and $\psi(\boldsymbol{\varepsilon}(\boldsymbol{u}), d)$ is the strain energy stored per unit volume of the material. Here, the value of ψ depends not only on the displacement $\boldsymbol{u}(\boldsymbol{x})$ but also on the crack phase field $d(\boldsymbol{x})$. Now, we turn our attention to the constitutive assumptions concerning the degradation of strain energy and stiffness that are directly related to the driving force of crack propagation. Depending on the constitutive assumptions regarding strain energy degradation, there are two major formulations, namely, isotropic and anisotropic. More detailed

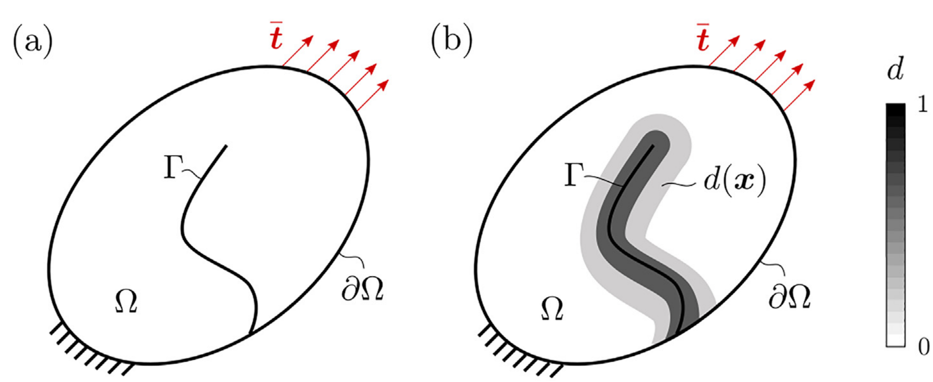

Fig. 1. Two-dimensional crack topology: (a) sharp crack model, (b) diffusive crack model described by phase field function $d(\boldsymbol{x})$.