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Analysis of time-dependent failure rate and probability of nuclear component

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ABSTRACT

Reliability data of nuclear component are the input parameter and important to the nuclear power plant probabilistic safety assessment. If the failure is weariness in origin, the failure rate and probability will be not constant and have a time trend. Failure rate and probability have been proved to be increasing with time by reliability mathematics. The generalized linear model is built for Poisson and Binomial distribution and applied to study the time-dependent failure rate and probability of nuclear component. The model is proved to be reasonable by the qualitative graph and Bayesian chi-square statistic methods, which can predict the real time trend of nuclear component failure rate and probability correctly. Only analyzing the time dependent failure rate and probability of nuclear component, we will build more accurate general reliability data to analyze the uncertainty of system reliability.

1. Introduction

Reliability data of nuclear component are important to nuclear power plant probabilistic safety assessment (PSA) and have a basic significance to guarantee the quality and reasonable results of PSA model. If the component reliability data of the power plant are used directly, there may exist the problem of little sample space. If the general database is used, it may not reflect the component reliability characteristics of the power plant. In the past years, Bayesian method was employed widely to update the general database through "new evidences" observation failure data of component by the nuclear power plant PSA, and obtain suitable component reliability data for the nuclear power plant PSA model (Dana and Curtis, 2011; Atwood, 1996). Bayesian method not only eliminates the drawbacks that the general database can't represent characteristics of the power plant, but also makes up the defect of a little of certain data and a great uncertainty. So Bayesian method was suggested to determine the reliability parameters by American Nuclear Regulatory Commission (NRC) and American Society of Mechanical Engineering (ASME) for building the reliability general database (Eide et al., 2007; ASME/ANS RA-Sa-2009; Atwood et al., 2003).

Currently, Jeffreys prior has been as the non-informative prior method in processing the nuclear component reliability data. Antonio and Fabrizio (2004) introduced Bayesian method to the reliability analysis of complex repairable systems. Several authors built Jeffreys prior models of Binomial and Poisson distribution on the basis of constant failure rate and probability, but they didn't analyze the time trend of failure rate and probability (He and Zhang, 2013; Shen et al., 2014).

The working nuclear components will exceed the tolerance limit of material and fail due to the cumulative damage in the temperature, mechanical, water, and so on. The failure comes from the weariness, and failure rate and probability aren't constant and relate to time. It is necessary to study the time-dependent problem of nuclear component failure rate and probability, so that we can obtain the time trend of component failure rate and probability correctly. Tan et al. (2005) used the homogeneous Poisson process and Laplace test to analyze the time-dependent failure rates of large-size generating units. Dai et al. (2010) analyzed the variable incidence with negative Binomial distributions. At present, the published literature takes little account of the time-dependent failure rate and probability of nuclear component.

In this study, the inherent characteristics of nuclear component, in which the failure data obey Binomial and Poisson distributions, are analyzed profoundly. The corresponding Bayesian Generalized Linear Model (GLM) is built to analyze the time trend of component failure rate and probability, and two cases are introduced to validate the model correctness. The database is established to analyze the uncertainty of system reliability through time trend of

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component failure rate and probability, which also can predict the component failure probability correctly.

2. Necessity to study time trend for component failure rate and probability

Nuclear components will fail to work due to the damaged factors such as the impact, vibration, abrasion, corrosion and so on. Some failure data obey the Binomial distribution and their demanding failure probability relates to time, and others obey Poisson distribution and their operating failure rate relates to time. In this section, we analyze the necessity to study time trend for component failure rate and probability.

2.1. Time trend for component failure rate

When environmental stress impacts the nuclear component as Poisson distribution, the component will have three characteristics as follows:

1) Probability of k impacts has nothing to do with time initially and is approximately proportional to the length of the time interval. The constant of proportionality is denoted by lambda (λ).
2) The occurrence of an event in one time interval does not affect the probability of occurrence in another non-overlapping time interval.
3) The probability of simultaneous events in a short time interval is approximately zero.

It is assumed that the components impacted by above environmental stress have x_i failure number in operation time t_i, the probability will be given by the Poisson (λt_i) distribution as follows:

$$P(x_i|\lambda) = \frac{(\lambda t_i)^{x_i} e^{-\lambda t_i}}{x_i!}, \quad x_i = 0, 1, \cdots, \quad n, i = 0, 1, \cdots, m, \quad \lambda > 0 \tag{1}$$

where t_i is the operation time, x_i is the failure number, λ is the failure rate and i is the number of operation time. The reliability and failure function of components are given as follows, respectively

$$\begin{cases} R(k) = \sum_{j=k}^{n} p_j \\ \lambda(k) = \frac{p_k}{\sum_{j=k}^{n} p_j} \end{cases} \quad j = 0, 1, \cdots n \tag{2}$$

$r = \frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}}$, and the kind of λ is measured by r.

$$\frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}} \begin{cases} > 0, & \text{increasing } \lambda \\ = 0, & \text{constant } \lambda \\ < 0, & \text{decreasing } \lambda \end{cases} \tag{3}$$

The formula used to determine the kind of λ for Poisson distribution is given by

$$r = \frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}} = (\lambda t)^j \frac{jk!}{(k+j+1)!} > 0 \tag{4}$$

From Eq. (4) it is seen that the kind of λ for Poisson distribution is increasing, so it is necessary to analyze time trend of failure rate λ which can reflect time-dependent failure rate of component correctly.

2.2. Time trend for component failure probability

The Binomial distribution is often used to describe the demand failure of nuclear component, for example, a valve doesn't change the state in response to a demand. Components which failure data obey Binomial distribution have following characteristics:

1) There are two possible outcomes of each demand: success and failure, components can and can't change the state in response to a demand.
2) There is a constant probability (p) of failure/success on each demand.
3) The result of each demand is independent, the outcomes of earlier demands do not influence that of later demands (i.e., the order of failures/successes is irrelevant).

Assuming the failure probability is p and the success probability is 1-p, there are n demands for components in an experiment and m experiments, random variable X is the failure number, $x = (x_1, \cdots, x_n)$ is the sample of X with Binomial distribution and its distribution function is given as follows:

$$\begin{aligned} p_{x_i} &= P(x_i|p) & x_i &= 0, 1, \cdots, n \\ &= C_n^{x_i} p^{x_i} (1-p)^{n-x_i} & i &= 1, 2, \cdots, m \end{aligned} \tag{5}$$

where i is the experiment number, x_i is the failure number in ith experiment, $C_n^{x_i}$ is the Binomial coefficient and p_{x_i} is the probability with x_i in n demands. The reliability and failure function of components are given by Eq. (2), $r = \frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}}$, the kind of failure probability p is measured by r.

$$\frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}} \begin{cases} > 0, & \text{increasing } p \\ = 0, & \text{constant } p \\ < 0, & \text{decreasing } p \end{cases} \tag{6}$$

The formula used to determine the kind of p for Binomial distribution is given by

$$\begin{aligned} r &= \frac{p_{k+j}}{p_k} - \frac{p_{k+j+1}}{p_{k+1}} \\ &= \frac{(k+1)!(n-k)!p^j}{(k+j+1)!(n-k-j)!(1-p)^j} \left[\frac{k+j+1}{k+1} - \frac{n-k-j}{n-k} \right] > 0 \end{aligned} \tag{7}$$

From Eq. (7) it is seen that the kind of p for Binomial distribution is increasing, so it is necessary to analyze time trend of failure probability p.

3. Time-dependent failure rate of nuclear component

Time-dependent failure rate of nuclear component which failure data obey Poisson distribution is analyzed in this section.

3.1. Jeffreys prior model: Poisson-Gamma model

Jeffreys prior model of Poisson-Gamma is obtained based on Bayes theorem. Second derivative of logarithmic likelihood function for Eq. (1) is given by

$$\frac{d^2 l(\lambda|x_i)}{d\lambda^2} = \frac{d^2 \sum_{i=1}^{n} \ln(P(x_i|\lambda))}{d\lambda^2} = -\frac{1}{\lambda^2} \sum_{i=1}^{n} x_i \tag{8}$$

where $l(\lambda|x_i)$ is logarithmic likelihood function of Poisson distribution. According to Eqs. (1) and (8), Fisher information matrix of parameter λ for Poisson distribution is

$$I(\lambda) = E\left(-\frac{d^2 l(\lambda|x_i)}{d\lambda^2}\right) = E\left(\frac{1}{\lambda^2} \sum_{i=1}^{n} x_i\right) = \frac{nt_i}{\lambda} \tag{9}$$