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Research articles

# Experimental error analysis of measuring the magnetic self-levitation force experienced by a permanent magnet suspended in magnetic fluid with a nonmagnetic rod

Jun Yu[a,b,*], Jiawei Chen[a,b], Decai Li[a,c,*]

[a] School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing 100044, China
[b] Key Laboratory of Vehicle Advanced Manufacturing, Measuring and Control Technology, Ministry of Education, China
[c] State Key Laboratory of Tribology, Tsinghua University, Beijing 100084, China

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ABSTRACT

The self-levitation of permanent magnet in magnetic fluid has been widely used in sensors, actuators, dampers, bearings, and lubrication in micromechanical systems. In this paper, the magnetic self-levitation force (MSLF) experienced by a permanent immersed in magnetic fluid is studied, and the formula for calculating the MSLF is derived. To measure MSLF experimentally, a nonmagnetic rod is used to connect the magnet and dynamometer. The experimental error caused by the nonmagnetic rod is studied theoretically, and nonmagnetic Cu rods with different radii are used in the experiments to verify the theoretical analysis. With nonmagnetic Cu rod sticking on the top surface of the magnet, the experimental error caused by the nonmagnetic rod is negligible when the magnet moves down from center of the container. However, the experimental error can't be ignored when the magnet moves up. In addition, the simulation results are in good agreement with the experimental results when the magnet moves down.

## 1. Introduction

Magnetic fluid (ferrofluid) is a new type of functional material with fluidity and magnetism, it is composed of ferromagnetic particles of subdomain size, surfactant and liquid carrier. The ferromagnetic particles coated with surfactant are evenly dispersed in the liquid carrier, forming a stable colloidal solution. The fluidity and magnetism of magnetic fluid have been exploited in many applications, such as sealing [1,2], dampers [3–5], sensors and actuators [6–9], bearings [10], position systems [11], lubrication in micromechanical systems [12–14] and so on.

This work concerns the magnetic levitation force experienced by a permanent magnet immersed in magnetic fluid. The magnetic levitation force which is called buoyant levitation of the second kind (also known as magnetic self-levitation force in this paper) was mentioned for the first time by Rosensweig [15,16] in 1966. However, due to the fact that it is a tedious work to calculate the magnetic field generated by a magnet, the research mainly focuses on the magnet with symmetric structure, such as spherical and cylindrical magnets.

Kvitantsev et al derived the magnetic self-levitation force (MSLF) acting on a spherical magnet immersed in a spherical container filled with magnetic fluid in [17] and obtained the trajectories of the spherical magnet in [18]. In [19], Yang et al used magnetic charge image method, current image method and finite element method to calculate the MSLF acting on a cylindrical magnet. Yu et al [20] derived the expression for calculating the MSLF acting on a magnet regardless of the shape of the magnet, and used three methods to calculate the MSLF acting on a cylindrical magnet.

For the measurement of MSLF, there were some methods proposed previously by some researchers. For example, He et al utilized a thin wire to bind a permanent magnet to measure the MSLF acting on the permanent magnet in [19] and [21]. In [20] and [22], nonmagnetic rods were used to connect the magnet and dynamometer, and the MSLF was measured by the dynamometer. In [23], Yu et al measured MSLF through the measurement of the magnetic force acting on the magnetic fluid under the magnetic field generated by the magnet.

However, for the method to measure the MSLF with a thin wire as mentioned in [21] and [19], the MSLF can't be measured effectively in the region where the MSLF is greater than the gravity of the magnet. For the novel method mentioned in [23], the experimental error caused by the magnetic shielding effect of magnetic fluid and the movement of magnetic fluid under nonuniform magnetic field is unavoidable, the

* Corresponding authors at: School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing 100044, China.
*E-mail addresses:* 16116356@bjtu.edu.cn (J. Yu), dcli@bjtu.edu.cn (D. Li).

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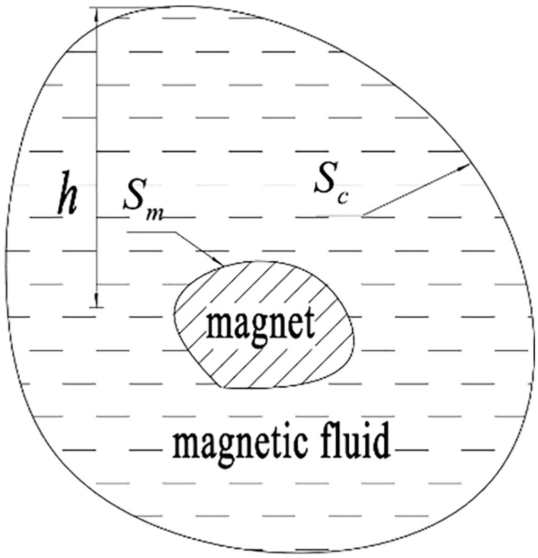


**Fig. 1.** A permanent magnet suspended in a container filled with magnetic fluid.

experimental results of MSLF will be greater than the calculation results. For the method mentioned in [20] and [22], the experimental error caused by the nonmagnetic rods hasn't been studied experimentally.

In this paper, we use nonmagnetic rods to connect the permanent magnet and dynamometer for the measurement of the MSLF experienced by the permanent magnet. The formula for calculating the MSLF is derived through which the theoretical model for experimental error caused by the nonmagnetic rods is established. In the experiment of measuring MSLF, nonmagnetic rods with different radii are used to study the experimental error caused by the nonmagnetic rods. The effect of the nonmagnetic rods on the MSLF has been worked out by theoretical analysis and experimental verification, respectively.

## 2. Theoretical analysis

As shown in Fig. 1, there is a permanent magnet suspended in magnetic fluid. In static state, the pressure of magnetic fluid can be written as [24]

$$p = p_g + p_s + p_m \tag{1}$$

The pressure generated by gravity is

$$p_g = \rho g h \tag{2}$$

The magnetostrictive pressure can be given as

$$p_s = \mu_0 \int_0^H \left(\nu \frac{\partial M}{\partial \nu}\right)_{H,T} dH \approx 0 \tag{3}$$

The fluid-magnetic pressure can be expressed as

$$p_m = \mu_0 \int_0^H M dH \tag{4}$$

where $\rho$ is the density of the magnetic fluid, $\nu = 1/\rho$, $g$ is the magnitude of gravity acceleration, $H$ is the magnitude of magnetic field, $M$ is the magnitude of magnetization of the magnetic fluid, $T$ is the absolute temperature, $\mu_0$ is the permeability of vacuum and $h$ is the height measured vertically below the top surface of the container.

According to the divergence theorem, for the scalar quantity $p$, we have

$$\oint_S p d\boldsymbol{a} = \oint_{S_c+S_m} p d\boldsymbol{a} = \int_{V_0} \nabla p dv_0 \tag{5}$$

where $V_0$ is the volume of the magnetic fluid, the boundary of which we shall denote by $S$ or $S_c + S_m$, $S_c$ is the inner surface of the container and $S_m$ is the surface of the magnet, $\boldsymbol{a}$ is the infinitesimal vector whose magnitude is the area of the small element of $S$ with direction outward-magnetic-fluid-pointing normal to that little patch of surface, $dv_0$ is the volume element.

The gradient of $p$ can be described as

$$\nabla p = \nabla\left(\mu_0 \int_0^H M dH + \rho g h\right) = \mu_0 M \nabla H + \rho \boldsymbol{g} \tag{6}$$

The right side of Eq. (5) can be written as

$$\int_{V_0} \nabla p dv_0 = \int_{V_0} \mu_0 M \nabla H dv_0 + \int_{V_0} \rho \boldsymbol{g} dv_0 \tag{7}$$

The left side of equation (5) is

$$\oint_S p d\boldsymbol{a} = \oint_S p_g d\boldsymbol{a} + \oint_S p_m d\boldsymbol{a} \tag{8}$$

Combining Eqs. (7) and (8), Eq. (5) becomes

$$\oint_S p_g d\boldsymbol{a} + \oint_S p_m d\boldsymbol{a} = \int_{V_0} \mu_0 M \nabla H dv_0 + \int_{V_0} \rho \boldsymbol{g} dv_0 \tag{9}$$

By Eq. (9), we have

$$\oint_S p_g d\boldsymbol{a} = \int_{V_0} \rho \boldsymbol{g} dv_0 \tag{10}$$

and

$$\oint_S p_m d\boldsymbol{a} = \int_{V_0} \mu_0 M \nabla H dv_0 \tag{11}$$

We write the gravity of magnetic fluid as

$$\boldsymbol{F_g} = \int_{V_0} \rho \boldsymbol{g} dv_0 = \oint_S p_g d\boldsymbol{a} \tag{12}$$

The magnetic force acting on the magnetic fluid under the magnetic field generated by the magnet is

$$\boldsymbol{F_m} = \int_{V_0} \mu_0 M \nabla H dv_0 = \oint_S p_m d\boldsymbol{a} \tag{13}$$

The boundary of the magnetic fluid is composed of $S_c$ and $S_m$. Eq. (11) can also be expressed as

$$\oint_{S_c} p_m d\boldsymbol{a} = \int_{V_0} \mu_0 M \nabla H dv_0 - \oint_{S_m} p_m d\boldsymbol{a} \tag{14}$$

The MSLF experienced by the magnet can be written as

$$\boldsymbol{F_{II}} = -\int_{V_0} \mu_0 M \nabla H dv_0 + \oint_{S_m} p_m d\boldsymbol{a} = -\oint_{S_c} p_m d\boldsymbol{a} \tag{15}$$

From Eq. (15), we can conclude that the MSLF can be calculated by integrating $p_m$ over $S_c$. To verify Eq. (15) and simplify the calculation, a cylindrical magnet and a cylindrical container are used in experiments as shown in Fig. 2(a). The axis of the container coincides with the that of the magnet, and $S_c$ becomes

$$S_c = S_{c1} + S_{c2} + S_{c3} \tag{16}$$

where $S_{c1}$, $S_{c2}$ and $S_{c3}$ are the bottom, side and top surface of the cylindrical container, respectively.

The integral of $p_m$ over $S_{c2}$ must be zero, we have

$$\int_{S_{c2}} p_m d\boldsymbol{a} = 0 \tag{17}$$

Therefore, combining Eqs. (15)–(17) gives

$$\boldsymbol{F_{II}} = -\int_{S_{c1}} p_m d\boldsymbol{a} - \int_{S_{c3}} p_m d\boldsymbol{a} \tag{18}$$

To measure the MSLF experimentally, a nonmagnetic rod is used to connect the magnet and dynamometer, and the cylindrical nonmagnetic