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Coupling magneto-elastic Lagrangians to spin transfer torque sources

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ABSTRACT

The consequences of coupling magnetic and elastic degrees of freedom, where spins and deformations are carried by point-like objects subject to local interactions, are studied, theoretically and by detailed numerical simulations. From the constrained Lagrangians we derive consistent equations of motion for the coupled dynamical variables. In order to probe the dynamics of such a system, we consider external perturbations, such as spin transfer torques for the magnetic part, and homogeneous stresses for the elastic part, associated to their corresponding damping. This approach is applied to the study of ultrafast switching processes in anti-ferromagnetic systems, which have recently attracted attention as candidates for anti-ferromagnetic spintronic devices. Our strategy is then checked in simple, but instructive, situations. We carried out numerical experiments to study, in particular, how the magnetostrictive coupling and external stresses affect the nature of the switching processes in a prototype anti-ferromagnetic material.

Introduction

The simplest classical field theory to describe the consequences of local interactions between magnetic and mechanical degrees of freedom is set up and its consequences are studied by numerical methods.

The starting point is a single, point-like, object carrying both, a classical spin vector, and a mechanical strain tensor, which can both depend on time. Early attempts may be found in many references [1–3].

In the canonical formulation, one has to consider the Lagrangian functional density $\mathscr{L}$ as a sum of three main contributions: The first one is the magnetic part, labeled $\mathscr{L}_s$, a functional of both a vector $\boldsymbol{s}(t)$ and its velocity $\dot{\boldsymbol{s}}(t)$. Here the classical spin (or magnetic moment), i.e. the vector $\boldsymbol{\mu}(t)$, is to be identified with $\dot{\boldsymbol{s}}(t)$ instead of $\boldsymbol{s}(t)$ [4].

This can be explained as follows: As there is no point–like "magnetic charge", in order to deduce an equation for the spin precession, that is second order in time, the potential vector has to depend on the history of the variable $\boldsymbol{s}(t)$, hence it is non-locally dependent on it. Another point of view would be to consider a "magnetic monopole", but such considerations, that lead to so many implications beyond the classical level of description we want to address, will not be discussed here [5].

The second one is the mechanical part, labeled $\mathscr{L}_m$, a functional of the symmetric Cauchy strain tensor $\epsilon_{ij}(t)$ and its time derivative $\dot{\epsilon}_{ij}(t)$. It represents a first approximation of what would be a dynamical Hooke's law. This viscoelastic approach is the starting point of studies of mechanical dynamical deformations in materials [6].

Finally, there is the coupling between these two systems, labeled by $\mathscr{L}_{sm}$ and commonly called "magnetostriction", in this context [7].

More precisely, these Lagrangians are given by the expressions:

$$\mathscr{L}_s = \frac{m_s}{2}\dot{s}_i^2 + \dot{s}_i A_i[\boldsymbol{s}] - V_s[\boldsymbol{s}] \tag{1a}$$

$$\mathscr{L}_m = \frac{m_\epsilon}{2}\dot{\epsilon}_{ij}^2 - V_\epsilon[\epsilon] \tag{1b}$$

$$\mathscr{L}_{sm} = -\frac{1}{2}B_{ijkl}\dot{s}_i\dot{s}_j\,\epsilon_{kl} \tag{1c}$$

These can be understood as describing interacting objects. One is a point–like particle, whose position is labeled by $s_i(t)$. The other is, in fact, an extended object, whose "position" is $\epsilon_{ij}(t)$. Latin indices run from 1 to 3, and the Einstein summation convention of repeated indices is assumed.

The Lagrangian $\mathscr{L}_s$ is invariant under local $U(1)$ transformations, i.e. $\delta A_i = \partial_i f(\boldsymbol{s})$, $\delta s_i = 0$, since the Lagrangian changes by a total derivative [8].

The first particle couples to the vector potential $\boldsymbol{A}[\boldsymbol{s}]$, which describes a physical magnetic field–however, since it is only magnetically charged, it couples through its gyromagnetic ratio.

Because $\dot{\boldsymbol{s}}$ represents the spin variable, m_s is an inertia constant which is here to describe the precession and may be interpreted as a Landé factor, V_s is a scalar potential, that gives rise to an "electric field" which can affect the conservation of the norm of the magnetization vector. By pursuing the analogy with the charged particle in an electromagnetic field, $\boldsymbol{A}[\boldsymbol{s}]$ is a vector potential, which depends on the whole history of $\boldsymbol{s}(t) = \int_0^t \dot{\boldsymbol{s}}(\tau)d\tau$ and, as remarked above, transforms under $U(1)$.

The elastic medium is considered spatially uniform and the second

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Lagrangian describes the deformation of the elastic medium [9]. Eq. (1b) means, in particular, that $\mathscr{L}_{\rm m}[\epsilon]$ defines a matrix model so the trace operation is implicitly assumed. Moreover, if the elastic medium is isotropic, this term is invariant under local $SO(3)$ transformations, that act with the adjoint action: $\epsilon_{ij} \to [R \epsilon R^{\rm T}]_{ij}$, with $R \in SO(3)$; so the full symmetry group of the theory, without interaction between particles, is $U(1) \times SO(3)$.

In the expression of $\mathscr{L}_m$, m_ϵ is an inertia term for the mechanical part. V_ϵ represents a scalar mechanical potential and can be expressed in an elastic medium as $V_\epsilon = \frac{1}{2} C_{ijkl} \epsilon_{ij} \epsilon_{kl}$ where C is the elastic stiffness tensor. Associated to this tensor, there is an elastic compliance tensor S such that $C_{ijkl} S_{ijmn} = \frac{1}{2}(\delta_m^k \delta_n^l + \delta_n^k \delta_m^l)$.

Finally, for $\mathscr{L}_{\rm sm}$, B_{ijkl} is a coupling matrix responsible for magnetostriction which is taken independent of the dynamical variables [7]. For the interaction term to be, also, invariant under $U(1) \times SO(3)$, the fields, s_i and ϵ_{ij} must carry "charges" that are related in a quite specific way [8,10]. In the case at hand, the invariance of the Lagrangian $\mathscr{L}_{\rm sm} = \dot{s}_i \dot{s}_j {\rm Tr}_{SO(3)}[B_{ijkl} \epsilon_{kl}]$ requires that B transforms itself as $B_{ijkl} \to [R^T B R]_{ijkl}$ with the proper selections of indices.

In all these expressions the indices are "space–like" and an immediate question is, whether the rotational symmetry thus implied can be promoted to a full–fledged, emergent, Lorentz symmetry. It is here that the "no–interaction theorem" [11] is relevant and implies that this is not possible, with a fixed–here two–number of particles (or for a matrix of fixed, finite, rank, referring to the ϵ_{ij}). This means, in particular, that, even if both inertia coefficients, m_s and m_ϵ, vanish, the excitations are not, in fact, massless, since the emergent Lorentz invariance is not compatible with any interaction term. How Lorentz invariance can emerge in such systems is, currently, the subject of considerable activity–but the constraints from the no–interaction theorem seem not to have been fully appreciated and deserve further investigation. In the following we shall work out some of the consequences of the $U(1) \times SO(3)$ symmetry as acting on the spatial indices.

In order to probe the dynamics of all the internal system variables, external sources are necessary. These sources can–and here will be assumed to–couple minimally to the fields and give rise to force terms in the equations of motion.

For forces that can be expressed in terms of scalar potentials, we have $\mathscr{L}_{sources} = -j_i^{\rm ext}[\boldsymbol{s}] \dot{s}_i - \sigma_{ij}^{\rm ext} \epsilon_{ij}$. At this step, regarding the magnetic part, $\boldsymbol{j}^{\rm ext}[\boldsymbol{s}]$ is a conserved current and cannot give rise to a spin transfer torque (STT). $\boldsymbol{\sigma}^{\rm ext}$ is an external, spatially uniform and instantaneous mechanical stress tensor. Extensions to non-instantaneous and non-uniform sources do not present any conceptual difficulties [12].

In order to derive expressions for the dissipative contribution in the Lagrangian formalism, one can remark that Gilbert's dissipation functions for spins and STT can be mapped to currents, when they are not functions of $\boldsymbol{s}$ only, but also of higher order time derivatives such as:

$$\frac{\partial \mathscr{L}_{\rm losses}}{\partial \ddot{s}_i} = \alpha\, \epsilon_{ijk}\, \dot{s}_j \ddot{s}_k + J(\dot{s}_i \dot{s}_j p_j - p_i \dot{s}_j \dot{s}_j) \tag{2}$$

where J is the amplitude of the current and $\boldsymbol{p}$ its direction. As expected, the sign of the spin-torque dissipation function depends, apart from the direction of the current flow, on the relative magnetization configuration of the magnetic layers.

Using the same kind of reasoning, the elastic current σ_{ij} can be decomposed into two terms

$$\sigma_{ij} = \sigma_{ij}^{\rm ext} - \gamma\, \dot{\epsilon}_{ij} \tag{3}$$

where $\sigma_{ij}^{\rm ext}$ are the components of an external applied stress tensor, which derive from a potential energy function, and γ is a mechanical damping constant, which is proportional to the strain time rate.

For each dynamical variable, Euler–Lagrange equations of motions (EOM)

$$\frac{\partial \mathscr{L}}{\partial s_i} - \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{s}_i}\right) = \frac{\partial \mathscr{L}_{sources}}{\partial \dot{s}_i} + \frac{\partial \mathscr{L}_{losses}}{\partial \ddot{s}_i} \tag{4a}$$

$$\frac{\partial \mathscr{L}}{\partial \epsilon_{ij}} - \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\epsilon}_{ij}}\right) = \frac{\partial \mathscr{L}_{sources}}{\partial \epsilon_{ij}} + \frac{\partial \mathscr{L}_{losses}}{\partial \dot{\epsilon}_{ij}} \tag{4b}$$

take the form

$$m_s \ddot{s}_i + F_{ij} \dot{s}_j + \frac{\partial V_s}{\partial s_i} - B_{ijkl}(\ddot{s}_j \epsilon_{kl} + \dot{s}_j \dot{\epsilon}_{kl}) = j_i \tag{5a}$$

$$m_\epsilon \ddot{\epsilon}_{ij} + \frac{\partial V_\epsilon}{\partial \epsilon_{ij}} + \frac{1}{2} B_{klij} \dot{s}_k \dot{s}_l = \sigma_{ij} \tag{5b}$$

where the antisymmetric Faraday tensor F is defined as usual:

$$F_{ij} = \frac{\partial A_i}{\partial s_j} - \frac{\partial A_j}{\partial s_i}$$

and describes spin precession, since it can be mapped to a dual pseudovector $\boldsymbol{\omega}$

$$F_{ij} \equiv \epsilon_{ijk}\, \omega_k[\boldsymbol{s}].$$

$\boldsymbol{\omega}$ is understood as the effective frequency of precession, and is usually defined as $\omega_i \equiv -\frac{1}{\hbar}\frac{\partial H}{\partial s_i}$, where H is the total spin hamiltonian, whose precise expression depends on the nature of the considered magnetic interactions.

The current $j_i = j_i^{\rm ext} + \frac{\partial \mathscr{L}_{losses}}{\partial \ddot{s}_i}$ is then the total torque applied on the spin system.

In more conventional terms, the bulk magnetization $\boldsymbol{M}(t)$, can be identified with the vector $Ng\boldsymbol{\mu}(t)/V$, where N is the number of magnetic moments, V is the volume and $g \equiv m_s$ the Landé factor. The magnetic induction $\boldsymbol{B}$ can be identified with the expression

$$\boldsymbol{B} = -\frac{1}{g\mu_B}\frac{\partial H}{\partial \boldsymbol{\mu}} \tag{6}$$

with μ_B is the Bohr's magneton. Finally, the magnetic field $\boldsymbol{H}$ can be defined by the relation between the magnetic induction and the magnetization

$$\boldsymbol{H} = -\boldsymbol{M} + \frac{\boldsymbol{B}}{\mu_0} \tag{7}$$

with μ_0 the permeability of the vacuum.

An advantage of our formulation is that these conventional quantities can be understood as emergent from a microscopic approach, that highlights the significance of the history of the sample. So in the following, we shall use the microscopic variables to describe the dynamics, since their relation to the conventional, macroscopic variables is transparent and allows a direct description of multisublattice effects, that have become of practical relevance and are much harder to unravel in terms of the macroscopic variables.

For it has been demonstrated that, as in ferromagnets, in multisublattice magnetic systems, also, the spin-polarized electrons transfer spin torques on each of the atomic sites [13–16]. Consequently, the magnetic structure of anti-ferromagnets (AFMs) may be described using "colored" vectors $\boldsymbol{s}^L$ and strain matrices ϵ_{ij}^L, that arise due to strong exchange magnetic coupling, where L labels the different inequivalent sites (or the sublattices).

The EOM take the form

$$m_s^L \ddot{s}_i^L + F_{ij} \dot{s}_j^L + \frac{\partial V_s}{\partial s_i^L} - B_{ijkl}(\ddot{s}_j^L \epsilon_{kl}^L + \dot{s}_j^L \dot{\epsilon}_{kl}^L) = j_i^L \tag{8a}$$

$$m_\epsilon^L \ddot{\epsilon}_{ij}^L + \frac{\partial V_\epsilon}{\partial \epsilon_{ij}^L} + \frac{1}{2} B_{klij} \dot{s}_k^L \dot{s}_l^L = \sigma_{ij}^L \tag{8b}$$

where $j_i^L = j_i^{\rm ext} + \alpha\, \epsilon_{ijk}\, \dot{s}_j^L \ddot{s}_k^L + J(\dot{s}_i^L \dot{s}_j^L p_j - p_i \dot{s}_j^L \dot{s}_j^L)$ and $\sigma_{ij}^L = \sigma_{ij}^{\rm ext} - \gamma\, \dot{\epsilon}_{ij}^L$.

Since the variable we are, really, interested in is $\boldsymbol{\mu}(t) \equiv \dot{\boldsymbol{s}}(t)$, we can rewrite the system as