Twists of quantum Borel algebras

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TWISTS OF QUANTUM BOREL ALGEBRAS

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Abstract. We classify Drinfeld twists for the quantum Borel subalgebra $u_q(\mathfrak{b})$ in the Frobenius-Lusztig kernel $u_q(\mathfrak{g})$, where $\mathfrak{g}$ is a simple Lie algebra over $\mathbb{C}$ and q an odd root of unity. More specifically, we show that alternating forms on the character group of the group of grouplikes for $u_q(\mathfrak{b})$ generate all twists for $u_q(\mathfrak{b})$, under a certain algebraic group action. This implies a simple classification of finite-dimensional Hopf algebras whose categories of representations are tensor equivalent to that of $u_q(\mathfrak{b})$. We also show that cocycle twists for the corresponding De Concini-Kac algebra are in bijection with alternating forms on the aforementioned character group.

1. Introduction

In this paper we classify Drinfeld twists for the quantum Borel subalgebra $u_q(\mathfrak{b})$ in the Frobenius-Lusztig kernel $u_q(\mathfrak{g})$, for a simple Lie algebra $\mathfrak{g}$ over $\mathbb{C}$ at an odd root of unity q. The algebra $u_q(\mathfrak{g})$ is also known as the *small quantum group*. We adopt some additional, minor, restrictions on the order of q which depend on the Dynkin type of $\mathfrak{g}$ (see Section 2).

Recall that Drinfeld twists for a given finite dimensional Hopf algebra H correspond to tensor structures on the forgetful functor from $\mathrm{rep}(H)$ to $Vect$. Here $\mathrm{rep}(H)$ denotes the tensor category of finite dimensional H-modules. Two twists are said to be gauge equivalent if their corresponding functors are naturally isomorphic. We let $\mathrm{Tw}(H)$ denote the set of gauge equivalence classes of twists for H. There is a group of *twisted autoequivalences* of H which acts on $\mathrm{Tw}(H)$, and the resulting quotient parametrizes isomorphism classes of Hopf algebras K which admit a tensor equivalence $\mathrm{rep}(K) \xrightarrow{\sim} \mathrm{rep}(H)$. (See Sections 3 and 8.1.)

For the small quantum Borel, it is known that the unipotent algebraic group $\mathbb{U}$, corresponding to the nilpotent subalgebra $\mathfrak{n} = [\mathfrak{b}, \mathfrak{b}] \subset \mathfrak{b}$, acts on the collection of twists for $u_q(\mathfrak{b})$ by way of twisted automorphisms [9, 30]. Basic considerations also establish an embedding $\mathrm{Alt}(G^\vee) \to \mathrm{Tw}(u_q(\mathfrak{b}))$, where $\mathrm{Alt}(G^\vee)$ denotes the set of alternating bilinear forms on the character group $G^\vee$ of the Cartan subgroup $G = G(u_q(\mathfrak{b}))$. We show below that the set of alternating forms on $G^\vee$ generates *all* twists for $u_q(\mathfrak{b})$ under the aforementioned action of $\mathbb{U}$.

Theorem A (8.2)**.** *There is an equality* $\mathrm{Tw}\,(u_q(\mathfrak{b})) = \mathbb{U} \cdot \mathrm{Alt}(G^\vee)$.

We also show in Proposition 6.3 that the De Concini-Kac algebra $U_q^{DK}(\mathfrak{b})$ admits no *cocycle* twists up to gauge equivalence, save for those coming from the group of grouplikes. A version of Theorem A can also be shown to hold for quantum Kac-Moody algebras. In this case one should complete $u_q(\mathfrak{b})$ relative to its grading by the root lattice.

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