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ACCEPTED MANUSCRIPT

GRADED DIVISION ALGEBRAS OVER THE FIELD OF REAL NUMBERS

YURI BAHTURIN AND MIKHAIL ZAICEV

Abstract. We give a full classification, up to equivalence, of finite-dimensional graded division algebras over the field of real numbers. The grading group is any abelian group.

1. Introduction

In this paper we will deal only with finite-dimensional algebras over a field F, which will be either the field $\mathbb{R}$ of real numbers or the field $\mathbb{C}$ of complex numbers. A unital algebra R over a field F graded by a group G is called *graded division* if every nonzero homogeneous element is invertible. Each such algebra is graded simple, that is, R has no proper nonzero graded ideals. As an ungraded algebra, a graded division algebra does not need to be simple, as shown by the basic example of the group algebra FG. But it is known (see, e.g. [8]) that graded division algebras are semisimple, that is, isomorphic to the sum of one or more simple algebras. According to the graded analogues of Schur's Lemma and Density Theorem (see, for example, [11] or [4] or [9]) any finite-dimensional graded simple algebra R is isomorphic to the algebra $\mathrm{End}_D V$ of endomorphisms of a finite-dimensional graded (right) vector space over a graded division algebra D. If, additionally, R is simple, it is obvious that D must be simple, as well.

In the case where the field F is algebraically closed, all simple graded division algebras have been described in [2] and [6]. For full account see [9, Chapter 1], where the authors treat also the case of Artinian algebras. In [3] (see also [7], for a particular case) the authors treat the case of graded primitive algebras with minimal one-sided graded ideals. If such algebras are locally finite, the graded division algebras arising by graded Schur's Lemma, are finite-dimensional and so the description provided in the case of finite-dimensional algebras works in this situation, as well.

In our recent paper [5] we have classified division gradings on simple real finite-dimensional algebras, up to equivalence. In [12] the author provided another approach to the classification of division gradings on these algebras, also up to isomorphism.

In the present paper we classify all finite-dimensional real graded division algebras. This is done case by case, depending on various factors. In Theorem 5.1 we deal with algebras endowed with so called Pauli or Sylvester gradings. They come

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