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# INFINITE DIMENSIONAL SERIAL ALGEBRAS AND THEIR REPRESENTATIONS

MIODRAG CRISTIAN IOVANOV

**ABSTRACT.** We study arbitrary algebras all of whose indecomposable finite dimensional representations are uniserial, or more generally, comodules over serial coalgebras. We show that the infinite representation theory for such algebras parallels and non-trivially extends the theory of primary infinite abelian groups: we obtain a Kulikov and a Prüfer decomposability criterion and determine indecomposables in the category of locally finite modules over such an algebra. As applications, we obtain consequences on the infinite representation theory of the extended affine Dynkin quiver  $\widetilde{\mathbb{A}}_n$ , and the infinite line and half line quivers, and obtain Jordan form results for operators on infinite dimensional spaces. We also find a class of counterexamples to an infinite version of the pure semisimplicity conjecture: there exist algebras all of whose locally finite left modules are direct sums of indecomposables, but not all of whose right locally finite modules decompose as such. On the way, we also provide classifications of one-sided serial (co)algebras, and introduce and classify a class of quantum groups whose indecomposable representations are uniserial, namely, coserial pointed Hopf algebras. These are one-parameter deformations of group algebras and are obtained as certain Ore extensions of these.

## INTRODUCTION

In the representation theory of finite dimensional algebras, as well as non-commutative ring theory, a central theme is the study of decompositions of modules into indecomposable objects. A particularly important and easier to understand class of indecomposable representations are the uniserial modules. Recall that a module over a ring is uniserial (or chain module) if the lattice of its submodules is a chain. A module is serial if it is a direct sum of uniserial modules, and a ring  $R$  is left serial if it is serial as a left module over itself.  $R$  is called serial if it is left and right serial. There are many structure theorems for serial rings especially for the case of noetherian and artinian rings (see for example [P]), and many results regarding modules over serial rings. If  $R$  is an artinian ring (and, in particular, for a finite dimensional algebra),  $R$  serial if and only if every left (equivalently, right) module decomposes as a direct sum of uniserial modules.

The class of finite dimensional serial algebras and that of artinian serial rings are also important as being among the first non-semisimple rings/finite dimensional algebras of finite representation type - that is, rings with only finitely many isomorphism types of indecomposable modules. Over rings of finite representation type, every left (or right) module decomposes into a direct sum of indecomposables. Conversely, if all left and all right modules over a ring  $R$  decompose as direct sums of indecomposables, then  $R$  is of finite representation type (these now classical results are due to Auslander, Fuller, Reiten, Ringel, Tachikawa; see [H-Z] for a detailed account). It is conjectured that decomposability only on one side is enough to imply finite representation type - the pure semisimplicity

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