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Characters of finite permutation groups and Krein parameters

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Abstract Let G be a transitive permutation group on a finite set Ω. If G is multiplicity-free, then $\mathrm{End}_G(\mathbb{C}[\Omega])$ is commutative, and Krein parameters $q_{i,j}^k$ can be defined. Scott proved that if $q_{i,j}^k \neq 0$, then the corresponding irreducible characters χ_i, χ_j, χ_k of G satisfy $(\chi_i\chi_j, \chi_k) \neq 0$. In this paper, we prove the converse of this implication for transitive permutation groups of semidirect product type whose regular normal subgroup is abelian.

Key words permutation groups, permutation characters, irreducible characters, commutative association schemes, Krein parameters

1 Introduction

A finite permutation group is called *multiplicity-free* if the permutation character is a sum of distinct irreducible characters. For a transitive permutation group G on a finite set Ω, let $\Lambda_0, \Lambda_1, \ldots, \Lambda_d$ be the orbits of G on $\Omega \times \Omega$, and $A_0, A_1, \ldots, A_d$ be the square matrices indexed by Ω such that $(A_i)_{x,y} = 1$ if $(x, y) \in \Lambda_i$ and 0 otherwise, for $i = 0, 1, \ldots, d$. Then the linear span $\mathcal{A} = \langle A_0, A_1, \ldots, A_d \rangle_{\mathbb{C}}$ is an algebra isomorphic to $\mathrm{End}_G(\mathbb{C}[\Omega])$, where $\mathbb{C}[\Omega]$ is the permutation module of G on Ω. If G is multiplicity-free, then $\mathcal{A}$ is commutative and the number of distinct irreducible characters appearing in the permutation character is equal to $d+1$. In other words, the permutation module $\mathbb{C}[\Omega]$ decomposes into $d + 1$ non-isomorphic irreducible G-modules: $\mathbb{C}[\Omega] = V_0 \oplus V_1 \oplus \cdots \oplus V_d$. For $i = 0, 1, \ldots, d$, let E_i be the orthogonal projection from $\mathbb{C}[\Omega]$ onto V_i. Then $E_i \in \mathcal{A}$ and $\{E_0, E_1, \ldots, E_d\}$ is a basis of $\mathcal{A}$. Let $\circ$ be the Hadamard product. Since $\mathcal{A}$ is closed under the Hadamard