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# Characters of finite permutation groups and Krein parameters

Keiji Ito

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**Abstract** Let  $G$  be a transitive permutation group on a finite set  $\Omega$ . If  $G$  is multiplicity-free, then  $\text{End}_G(\mathbb{C}[\Omega])$  is commutative, and Krein parameters  $q_{i,j}^k$  can be defined. Scott proved that if  $q_{i,j}^k \neq 0$ , then the corresponding irreducible characters  $\chi_i, \chi_j, \chi_k$  of  $G$  satisfy  $(\chi_i \chi_j, \chi_k) \neq 0$ . In this paper, we prove the converse of this implication for transitive permutation groups of semidirect product type whose regular normal subgroup is abelian.

**Key words** permutation groups, permutation characters, irreducible characters, commutative association schemes, Krein parameters

## 1 Introduction

A finite permutation group is called *multiplicity-free* if the permutation character is a sum of distinct irreducible characters. For a transitive permutation group  $G$  on a finite set  $\Omega$ , let  $\Lambda_0, \Lambda_1, \dots, \Lambda_d$  be the orbits of  $G$  on  $\Omega \times \Omega$ , and  $A_0, A_1, \dots, A_d$  be the square matrices indexed by  $\Omega$  such that  $(A_i)_{x,y} = 1$  if  $(x, y) \in \Lambda_i$  and 0 otherwise, for  $i = 0, 1, \dots, d$ . Then the linear span  $\mathcal{A} = \langle A_0, A_1, \dots, A_d \rangle_{\mathbb{C}}$  is an algebra isomorphic to  $\text{End}_G(\mathbb{C}[\Omega])$ , where  $\mathbb{C}[\Omega]$  is the permutation module of  $G$  on  $\Omega$ . If  $G$  is multiplicity-free, then  $\mathcal{A}$  is commutative and the number of distinct irreducible characters appearing in the permutation character is equal to  $d + 1$ . In other words, the permutation module  $\mathbb{C}[\Omega]$  decomposes into  $d + 1$  non-isomorphic irreducible  $G$ -modules:  $\mathbb{C}[\Omega] = V_0 \oplus V_1 \oplus \dots \oplus V_d$ . For  $i = 0, 1, \dots, d$ , let  $E_i$  be the orthogonal projection from  $\mathbb{C}[\Omega]$  onto  $V_i$ . Then  $E_i \in \mathcal{A}$  and  $\{E_0, E_1, \dots, E_d\}$  is a basis of  $\mathcal{A}$ . Let  $\circ$  be the Hadamard product. Since  $\mathcal{A}$  is closed under the Hadamard

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