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A population balance model of ball wear in grinding mills: An experimental case study

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ABSTRACT

A general and realistic population balance model is applied to a ceramic ball mill. The experimental data are obtained for three differently sized balls. The mill is operated 500 h with silica sand during 8 cycles. After each cycle, the mill is stopped to measure the ball wear to obtain the kinetics wear equations. It turns out that the wear law for all three different sized balls is of zero order. These experimental results are introduced into the model to obtain the ball charge of the mill at steady state and the alumina consumption by wear.

1. Introduction

Grinding consumes approximately 80% of the energy needed for the extraction of metallic and industrial minerals (Menacho, 1985; Austin and Concha, 1994; Fuerstenau and Abouzeid, 2007). This consumption is mainly determined by energy loss and wear of grinding media. Of all stages of mineral processing, grinding is the most inefficient, since only 1% of the energy supplied to the mill is effectively used for fracturing the mineral (Fuerstenau and Abouzeid, 2002). Therefore, the stage of fragmentation decisively determines the efficiency of plant operation.

Austin et al. (1984) and Austin and Concha (1994) proposed models of balance of balls in grinding mills. However, it remains unclear whether the charge must be done with balls of a unique size or with a distribution of balls of different sizes. In this particular article, we propose a charge of one size.

The analysis of the movement of the grinding media charge in mills dates back to a century ago, when Davis (1919) calculated the trajectories of a ball inside a rotatory mill, based on a simple balance of forces but neglecting friction. This simplification, added to the complexity of the problem due to the large number of particles (balls and mineral) present during the process, produced unsatisfactory results, and only Rose and Sullivan (1958) emphasized the need for considering friction in the corresponding calculations. The analysis of movement of the load in rotatory mills had been limited to calculations of the trajectories of a single ball for a long time when Powell and Nurick (1996) published the trajectories that should be followed by grinding media in a rotary ball mill, using a model which employs the concept of equilibrium surface.

All these results assume a purely mechanical viewpoint. They are useful for predicting the wear of balls and the size distribution of mineral particles within the mill, among other quantities. However, none of these works seems to consider the re-charge of balls. This shortcoming was inconvenient for the design of the grinding process. Bond (1960) was the first to model the relation between ball and particle sizes. Using a criterion based on the characterization of the size distribution at the entrance of the mill, he developed equations that allow one to select the ball sizes at the beginning of the operation, but little information about the recharge during operation is provided. In this sense, there is interest in studying the kinetic laws of wear of each grinding media, in particular to optimize the energetic consumption by wear, from population balances. Even today, the wear kinetics of grinding media is estimated based on theories developed in the first half of the 20th century. For instance, Sepúlveda (2004) used the theory of linear wear to calculate the constant of specific speed of wear.

From the point of view of the constitutive equations of the kinetics of wear of steel balls and their application within population balance models, the works by Menacho (1985) and Menacho and Concha (1986, 1987) constitute a significant advance since they were able to show that theoretical results compare well with the experimental ones. Menacho (1985) developed a phenomenological model of wear in rotatory mills starting from a population balance. With these results, it was possible to

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Nomenclature

$b(d)$	sieve function, defined in (6) [h^{-1}]
C_D^{SS}	ball consumption defined in (28) [$g\,h^{-1}$]
$C_{D,1}^{SS}$, $C_{D,2}^{SS}$	contributions to C_D^{SS} defined in s (28) [$g\,h^{-1}$]
d	ball diameter [mm]
d_k	size of balls of species k, $k = 1, ..., p$, fed into the mill [mm]
d_{ref}	reference ball diameter in wear law [mm]
d_{max}	maximal size [mm]
$g(d)$	rate of decrease in size of ball of diameter d [$mm\,h^{-1}$]
$H(\cdot)$	Heaviside function, defined in (4) [–]
J	fill factor [–]
m_F^k	relative number frequency of balls of size d_k in the feed [–]
$N(d,t)$	number density at time t [mm^{-1}]
$N_H(d,t)$	solution of the homogeneous problem [mm^{-1}]
$N_0(d)$	initial number density (size distribution) [mm^{-1}]
$N^{SS}(d)$	number density (size distribution) at steady state [mm^{-1}]
$Q_F(t)$	total number feed rate [h^{-1}]
$Q_S(t)$	total sieve rate of balls of a diameter less or equal d_0 [h^{-1}]
$\widetilde{Q}_S(t)$	scaled version of Q_S at steady state, defined in (24)
p	number of sizes $d_1, ..., d_p$ of new balls fed into the mill [–]
t	time [h]
$V(t)$	total volume of all balls at time t defined in (20) [mm^3]
V^{SS}	total volume of all balls at steady state (see (21)) [mm^3]
w_B^{SS}	total mass of all balls at steady state [mm^3]
$y_3(d,t)$	ball size mass frequency function defined in (19) [mm^{-1}]
$y_3^{SS}(d)$	ball size mass frequency function at steady state [kg]
α	negative constant in Eq. (2) [mm/h]
β	dimensionless exponent in Eq. (2) indicating wear law [–]
$\gamma(d_0,\beta)$	integral arising in the formula for V^{SS} defined in (23) [mm^4]
$\lambda(\cdot,\cdot)$	dimensionless auxiliary function defined in (10) [–]
$\chi_{[0,d_0]}(\cdot)$	characteristic function, defined in (5) [–]
ρ	density of balls [$g\,cm^{-3}$]
$\xi(\cdot,\cdot)$	function arising in the definition of $N_H(d,t)$ [mm]

describe the size distribution of grinding media in continuous mills operating in conditions of transient or stationary state, and to develop an optimization scheme of size profile of grinding media charge. These advances contribute to maximizing the efficiency of industrial circuits of grinding and classification. Bürger et al. (2005) improved the model proposed by Menacho and Concha (1986, 1987) so that it would produce positive solutions only. In fact, in its original formulation, the model proposed by Menacho and Concha (1986, 1987) produces negative solutions in some exceptional cases.

Population balance models were proposed first in chemical engineering by Hulburt and Katz (1964). They have turned out to be useful in a variety of applications including crystallization processes, grinding, agglomeration, flotation, solvents extraction, leaching, and polymer synthesis. We refer to Ramkrishna (2000), Verkoeijen et al. (2002) and Jakobsen (2008) for an introduction to population balance models of population balance and their applications and further references. In this article, a model of a simple population balance of the wear of ceramic balls in rotatory mills will be applied.

2. Materials and methods

The experimental work was performed by using a laboratory mill of diameter 570 mm and length 220 mm. The mill was filled with 54.84 kg of alumina balls of three different sizes (40.5 mm, 30.7 mm, and 26.9 mm) with a fill factor $J = 45\%$, and operating at 75% critical speed. A uniform ball size distribution of 33.3% of each size was used. The mill was loaded with 16.06 kg of monosized particles of silica sand with an average size of 0.093 mm and a bulk density of 2.601 g/cm^3. The diameters, weights, and density of the three ball types, as well as the volumes calculated from these quantities, can be seen in Table 1.

The charge consisting of ceramic balls and mineral was ground during 8 cycles that ended at the respective times $t = 90, 220, 292, 330, 390, 430, 470$, and 500 h. After each cycle, the mill was stopped and the wear (weight and diameter) was measured for every ball type. Before the milling process, 143 type I balls, 316 type II balls, and 498 type III balls were introduced into the mill. After every stop, the ball wear for each of the 143 type I balls was measured and the arithmetic mean was calculated. The same calculation was performed for the type II and III balls. The aim was to study ball size variations throughout the cycles. In Table 2 we show the equations obtained by the experimental setup for type I, II, and III balls. In particular, the coefficient of the linear term in the regression that approximates the decrease of size as a function of time provides a constant within the algebraic expression of the rate of decrease, denoted $g(d)$, of a ball with diameter d (as will be discussed in more detail in the next section). This result will be fundamental in order to obtain the ball entrance flux and ceramic ball wear. Typically, mill media wear rate is evaluated by measuring the media level in the mill, or by removing media charge and weighing it after a certain number of hours. As mentioned above, grinding tests were carried out for 21 days (500 h) with a total of 8 cycles for each ceramic ball type.

Table 1
Physical features of the alumina balls.

Ball Type	Density [g/cm^3]	Diameter [mm]	Mass [g]	Volume [cm^3]
Type I	3.6	40.5	127.93	34.7
Type II	3.6	30.7	57.78	15.1
Type III	3.6	26.9	36.67	10.2

3. Theory

3.1. Time-dependent population balance of wear of grinding media

The governing equation of the model by Bürger et al. (2005) is given by

$$\frac{\partial N(d,t)}{\partial t} + \frac{\partial}{\partial d}\left(g(d)N(d,t) - Q_F(t)\sum_{k=1}^{p} m_F^k H(d-d_k)\right) = -Q_S(t)\chi_{[0,d_0]}(d)N(d,t), \quad (1)$$

where d is the ball diameter and $g(d)$ is a function associated with the wear law. Here $\partial N(d,t)/\partial t$ (time derivative) is the local rate of change of $N(d,t)$, and the two terms under the spatial derivative $\partial/\partial d$ describe the decrease of $N(d,t)$ due to wear (as expressed by the term $g(d)N(d,t)$) and the increase due to feed entrance of balls of sizes $d_1, ..., d_p$ (as expressed by the term containing the sum; further explanations are provided below). The right-hand side of (1) is a sink term that describes the loss of balls (of size smaller or equal d_0) that are sieved out (again, see below for details).

The value of the function $g(d)$ is the rate of decrease of a ball of diameter d. The algebraic form of $g(d)$ depends on the materials under

Table 2
Kinetic equations.

Ball Type	Linear equation	R^2	$g(d)$ [$mm\,h^{-1}$]
Type I	$-0.25 \times 10^{-5}t + 40.8$	0.93795	-0.25×10^{-5}
Type II	$-0.13 \times 10^{-5}t + 31.2$	0.94578	-0.13×10^{-5}
Type III	$-0.12 \times 10^{-5}t + 27.0$	0.93981	-0.12×10^{-5}