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Nonlinear buckling analysis of variable stiffness composite plates based on the reduced order model

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ABSTRACT

The variable-stiffness fiber composite plates which have an enhanced design flexibility, largely rely on laminate optimizations to maximize the buckling performance. The corresponding computational efficiency becomes a key issue, in particular when the nonlinear structural behavior is considered. The finite element method based on a full nonlinear analysis is a standard technique for nonlinear structural analysis, however the high computational complexities generated from both the incremental-iterative procedure and the very refined mesh needed for the discrete modeling of curved fibers, are still a decisive cost factor on modern computers. In this work, the Koiter-Newton method is further extended to nonlinear buckling analysis, including the pre and post buckling stage, of variable stiffness composite plates. A four-node quadrilateral element based on the classical laminated plate theory is developed in framework of the *von Kármán* kinematics, for the finite element implementation of the proposed asymptotic method. The reduced order model, with or without imperfections, is constructed using the improved Koiter's asymptotic expansion, for both the symmetrical and unsymmetrical laminates. The nonlinear response curve of loaded structure can be traced automatically, using the nonlinear predictor and corrections both generated from the reduced order model. This leads to a fairly large step size in the path-tracing process, compared to that for the classical Newton method. The reduced order model largely reduces the computational burden produced by the high-density FE mesh for the varied fiber path. Numerical results indicate the overall high quality and efficiency of the proposed method.

1. Introduction

In the current design philosophy, the operating load carrying capability of thin-walled structures under axial or combined loads, can be driven by static buckling instabilities due to their favorable strength-to-weight ratio and their slenderness [1,2]. With the gradually increased demand of light-weight design of aerospace structures, advanced composites, which have some superior properties, such as light weight, high strength, good anti-fatigue property and environmental resistance, have gain more and more application in aviation and aerospace industries. In the presence of buckling or even before buckling, thin-walled composite structures often exhibit high out-of-plane displacements, compared to their wall thickness, which causes geometrically nonlinear structural responses [1,3]. Thus, nonlinear buckling analysis has been widely used to study the buckling and post-buckling behaviors of composite structures [3–5]. The buckling load-carrying capabilities of structures with variously functionally graded polymer composites have been investigated based on nonlinear buckling simulations [6–8]. Material failure criteria have been implemented into the buckling analysis of composite shells considering geometric nonlinearities [9,10]. Extensive studies [1,11–13] have indicated that geometric imperfections influence the nonlinear buckling behavior of composite structures.

The well-known advantages of utilizing fiber-reinforced laminated composites in structural design is the ability to change the stiffness and strength properties of the laminate by designing the laminate stacking sequence [13]. For the conventional straight-line fiber composite, the fiber orientation angle keeps constant within each layer throughout a structural component. To achieve a wider tailoring option in design, the concept of variable-stiffness fiber composite was proposed and developed recent years [14–19], where the fiber path is curvilinear and thus in-plane stiffness is spatially varying throughout the structure. Pre-defined mathematical expressions or their interpolation are commonly applied to prescribe the varied fiber path. Olmedo and Gürdal et al. [14] investigated a linear variation of fiber orientation angles for rectangular panels, which should be the earliest interpolation description work. Later, a plenty of work [16,20,21] has dedicated to developing the description of the fiber path, to further enhance the design space.

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The realization of varying fiber orientation angle makes variable stiffness composite plates have a higher design ability than straight-line fiber composite plate. In addition, variable-stiffness design has spatially varying bending and coupling stiffness, which is beneficial in achieving the in-plane and out-of-plane stiffness requirements. The buckling of thin-walled panels is accompanied by both a loss of in-plane stiffness and also growth of out-of-plane displacements. Thus, buckling performance can be considered as an important design constraint in the design process of thin and slender variable stiffness composite structures. Previous studies [14,21,22] demonstrate that a significant improvement of buckling loads can be achieved by using a variable-stiffness design. The improvement is attributed to the redistribution of in-plane loads to relatively stiff regions, and then resist buckling in critical regions. Since the buckling load is sensitive to the variable fiber configurations, numerous works [19,21,23–25] have been reported to maximize the buckling performance of variable-stiffness laminates using a optimized design of structure.

Previous works [14,15,21,24,25] on buckling optimization of variable-stiffness composite panels focused mainly on initial buckling load for bifurcation type instability, whereas few works [17,26] have been reported on the study of postbuckling behaviour or the limit-point type buckling, where structural geometrically nonlinearities are remarkable. The finite element method based on a full nonlinear analysis is a standard tool for nonlinear structural analysis [27]. Nevertheless, the high computational complexity generated from the incremental-iterative procedure for the refined finite element model, is still a decisive cost factor on modern computers. In addition, in the FEA framework, the curved fibers are discretized at each element and dealt with as straight fibers, for composite variable-stiffness panels [26,28]. This leads to a large prediction error of buckling behavior unless a very refined mesh size is applied, which severely increases the computational burden, especially for large-scale structures. The above computational power constraints become more severe in the optimization of variable-stiffness panels. The number of design variables have already grows rapidly with the increase of formulation complexity for fiber path, if structural geometrically nonlinearities with refined FE meshes are also considered into the optimized design, both the convergence rate and computational cost become very difficult to guarantee. Some analytic or semi-analytical techniques constitute very powerful tools for studying the nonlinear buckling behavior of variable stiffness composite structures. A semi-analytical variational approach based on the Rayleigh-Ritz method was developed by Wu et al. [17], to perform postbuckling analysis of variable stiffness composite plates under uniform axial compression loading. Later, they proposed an enhanced perturbation model for the moderately deep postbuckling analysis of rectangular composite plates under biaxial compression loading [29]. Raju et al. [30] presented a perturbation approach namely, asymptotic numerical method (ANM), to solve the postbuckling problem of variable stiffness composite laminates under axial compression, in the optimization framework. Rahman et al. [26] applied a finite element perturbation approach based on the Koiter perturbation theory to study the postbuckling behavior of variable stiffness fiber plates. A robust and computationally efficient method for analyzing the post-buckling behavior of flat variable-angle tow (VAT) plates has been proposed by Madeo et al. [31], using Koiter's asymptotic approach implemented in co-rotational FE framework. In addition, the buckling behavior of composite variable-stiffness panels is investigated by Hao et al. [32], based on isogeometric analysis, to guarantee the continuity of fiber angle on the whole panel.

In view of the high cost of computing usually required for nonlinear structural analysis, the attractiveness in reducing the problem size using the asymptotic methods [26,33–37] is obvious. However, the numerical accuracy of the methods is very sensitive to the selection of basis vectors, and Koiter's method is only valid for a small post-buckling range. In addition, analytic methods always lose potential for engineering applications, if the finite element implementation is unachievable or too complicated. To achieve a wider application, a novel hybrid approach, termed the Koiter-Newton method (KN) [38–41], has been proposed inspired by Koiter's initial post-buckling theory and Newton arc-length techniques, to trace the entire equilibrium path in a stepwise manner. The basic premise behind the approach is to use Koiter's asymptotic expansion at any equilibrium point from the beginning of the equilibrium path, rather than to use it only at the bifurcation point. In each incremental step, the method combines a prediction phase using a nonlinear reduced order model(ROM) based on improved Koiter asymptotic expansion with a Newton iteration based correction procedure, thus allowing the algorithm to use fairly large step sizes. The method is designed to be applicable to the numerical solution of a class of elastic nonlinear structural analysis problems, especially in the presence of buckling. The proposed Koiter-Newton technique requires derivatives of the strain energy with respect to degrees of freedom up to the fourth order. This is two orders more than traditionally needed for Newton's method. To facilitate the calculation of high-order derivatives, the element independent co-rotational (CR) kinematics have been applied together with the automatic differentiation technique [42,43]. Recently, the original Koiter-Newton method has been preliminarily improved to be the modified Newton-type Koiter-Newton method, which becomes more numerically robust and computationally efficient [44].

In this paper, the Koiter-Newton method is further extended to nonlinear buckling analysis of variable stiffness composite plates. A four-node quadrilateral element is developed based on the classical laminated plate theory, considering both the symmetrical and unsymmetrical laminates for variable stiffness composite plates. The *von Kármán* kinematics which possess an acceptable accuracy compared to the full nonlinear kinematics, is applied to facilitate the high-order derivatives of the elemental strain energy. The reduced order model with or without imperfections is constructed at any known equilibrium state and its solution will be treated as the nonlinear predictor for the response of the loaded structure. A novel corrector procedure is then applied on the nonlinear predictor, totally based on the reduced order model. The rank-one inverse Broyden quasi-Newton method is used to further improve the computational efficiency of the correction phase. The response curve, including nonlinear prebuckling and postbuckling stages, can be achieved efficiently in the above predictor-corrector phase using a step by step manner. The objective of this work is to provide the nonlinear buckling analysis of variable stiffness composite plates with an accurate and efficient numerical method. The finite element implementation makes the proposed method an practical method for complex engineering problems. The construction of the reduced order model largely reduces the computational burden generated from the very refined mesh for the modeling of varied fiber path.

The content of this paper is arranged as follows. The fundamental theory for variable-stiffness fiber composites is quickly reviewed in Section 2. A quadrilateral element for laminated composites is developed and the relevant finite element formulations needed for Koiter-Newton analysis are derived in Section 3. The Koiter-Newton method proposed for nonlinear buckling analysis of variable stiffness composite plates is presented in Section 4. Numerical examples which demonstrate the success of the method are provided in Section 5. In Section 6 we summarize the paper and draw conclusions.

2. Fundamental theory for variable stiffness composite plates

Traditional composite laminates usually consist of several plies, and the plies stack in a predefined order along the plate thickness, with an uniform fiber angle orientation throughout each ply. These laminates are referred to as constant stiffness laminates, since their stiffness properties are independent of spatial location. Variable stiffness laminates refer to laminates that consist of plies with continuously variable in-plane fibre orientations, and as a result, their stiffness properties are a function of spatial location, in other words stiffness properties change