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Stiffness matrices for fluid and anisotropic soil layers with applications in soil dynamics

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ABSTRACT

In this study, we introduce and discuss features and improvements of the well-established stiffness matrix method that is used in simulation of wave propagation in layered media. More specifically, we present stiffness matrices for an acoustic layer and a vertically transverse isotropic (VTI) viscoelastic soil layer. Combining these stiffness matrices enables a straightforward technique for modeling of acousto-elastic wave propagation in layered infinite media. In addition, we propose a technique to simulate discontinuity seismic sources, which was not used earlier in the context of the stiffness matrix method. Finally, we propose a framework to derive a key parameter of the absorbing boundary domain technique Perfectly Matched Layer (PML). Numerical examples are presented in order to help understanding the features and improvements discussed in the study from the fields of geophysics and soil dynamics. It is believed that the features and improvements discussed herein will make the application of the stiffness matrix method even wider and more flexible.

1. Introduction

The stiffness matrix method is a well-developed approach for simulating wave propagation in layered media, and has been successfully applied to various problems during the last decades (e.g. [1–3] and [4]). The method describes the wave motion in a layered medium in terms of symmetric and banded matrices and with straightforward and efficient solution procedure, producing the dynamic responses simultaneously at all layer interfaces and in all directions. The method has later been extended to acoustic layers ([5,6]). The discrete version of the stiffness matrix solution, called Thin-Layer Method (TLM), has also been developed and applied to various problems ([7–9]). Recently, TLM has been combined with the so-called Perfectly Matched Layer method (PML) that enables calculation of wave motion in infinite domains [10]. Despite these extensions, there are still features and improvements of the stiffness matrix method that could advance the use of the method in theoretical and applied problems. The present study introduces and discusses some of those features, including

- Vertically transverse isotropic (VTI) soil layer stiffness
- Discontinuity seismic sources
- Derivation of PML parameters

For completeness, first we present the acoustic layer stiffness matrices in forms that can be used in offshore or fluid-soil-coupled applications (e.g. seismic wave in the ocean environment). We introduce three different formulations in terms of vertical displacement, velocity potential and pressure. Each formulation has its own advantages and disadvantages. For example, the second and third formulations make it straightforward to implement the so-called air-gun source that is used as explosive acoustic source in offshore seismic surveys. This is because the air volume injected by the air-gun is explicitly defined in the two formulations, which are shown later. On the other hand, the first formulation is more suitable for applying vertical disk load on seabed or within water column, because the disk load can be represented by a term that can be set directly in the matrix equations. It is also shown that the three solutions are interrelated such that one can be derived from the other two through relevant constitutive laws. Next, the soil stiffness matrices for the vertically transverse isotropic (VTI) layers are derived for both P-SV (in-plane) and SH (anti-plane) wave modes. It is shown that the structure of the VTI soil layer stiffness matrices is identical to that of the isotropic soil layer, except that the parameters have different definitions and include the anisotropy factors (a and b). Indeed, the stiffness matrix for the isotropic case can be recovered by setting the anisotropy factors equal to 1. This allows straightforward extension of existing numerical tools based on stiffness matrices in isotropic soil to anisotropic soil. The derived stiffness matrices are used to compute the impedance matrices of square

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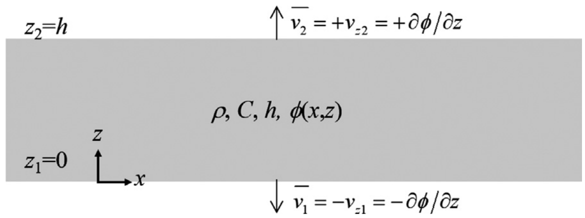

Fig. 1. Acoustic layer of mass density ρ, wave velocity C and thickness h whose motion is expressed by the particle velocity potential ϕ. Note that two flux boundary conditions at $z = 0$ and h are given in terms of volume change $\bar{v}$.

foundations on anisotropic soil media and the results are compared with their isotropic counterparts to highlight the effect of anisotropy on the foundation impedances. Further, in order to solve the case of injected (air or fluid) volume or dislocation/slip at the interface of two layers, we formulate a technique to implement displacement discontinuity into the stiffness matrix method. This technique enables the stiffness matrix method to simulate the wave fields generated by e.g. an air-gun source or dislocation seismic sources. Finally, by means of the continuum stiffness matrices we derive the key parameter of PML (i.e. PML thickness, h_{PML}) that can be used in discrete numerical approaches, for example, in TLM, Finite Element Method (FEM) and Finite Difference Method (FDM).

2. Stiffness matrices for fluid and anisotropic soil layers

2.1. Acoustic layer

Fig. 1 shows schematically an acoustic layer of thickness h. Wave motion in an acoustic layer can be described with different equations. In this study, we have chosen the particle velocity potential (ϕ). The governing equation in the space-frequency domain has then the following form:

$$\nabla^2\phi + \frac{\omega^2}{C^2}\phi = 0 \tag{1}$$

where ∇ is the Laplacian operator, ϕ is the velocity potential, ω is the angular frequency (in radian/s), and C is the wave velocity in the acoustic layer.

The velocity potential (ϕ) and the vertical velocity ($v_z=\partial\phi/\partial z$) in the wavenumber-frequency domain can be given in matrix forms as

$$\begin{Bmatrix}\phi\\ v_z\end{Bmatrix} = \begin{Bmatrix} e^{\beta z} & e^{-\beta z}\\ \beta e^{\beta z} & -\beta e^{-\beta z}\end{Bmatrix}\begin{Bmatrix}A\\ B\end{Bmatrix} \tag{2}$$

where A and B are unknown constants to be determined for each acoustic layer, $\beta = \sqrt{k^2 - \omega^2/C^2}$ is vertical direction wavenumber in the acoustic layer, and k is the radial (or horizontal) direction wavenumber. Note that hereby Fourier (for plane wave) or Hankel (for cylindrical wave) transformation from the spatial to wavenumber domain is already applied, and the wavenumber k replaces all the spatial derivatives with respect to the horizontal (or radial) coordinate. For an acoustic layer of finite thickness h, we can express explicitly the two quantities of the velocity potential and the vertical velocity on the top and bottom interfaces by setting $z = h$ and $z = 0$, resulting in the following two matrix equations.

$$\begin{Bmatrix}\phi_2\\ v_{z2}\end{Bmatrix} = \begin{Bmatrix} e^{\beta h} & e^{-\beta h}\\ \beta e^{\beta h} & -\beta e^{-\beta h}\end{Bmatrix}\begin{Bmatrix}A\\ B\end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix}\phi_1\\ v_{z1}\end{Bmatrix} = \begin{Bmatrix} 1 & 1\\ \beta & -\beta\end{Bmatrix}\begin{Bmatrix}A\\ B\end{Bmatrix} \tag{4}$$

Subscripts 1 and 2 indicate, respectively, the quantities on the bottom and top interfaces. By removing the unknown constant vector $\{A,\ B\}^T$ from the two matrix equations, we can obtain the following direct-relationship between the top and bottom interfaces (i.e. at $z = h$ and $z = 0$) in terms of the velocity potential and the vertical velocity given below

$$\begin{Bmatrix}\phi_2\\ v_{z2}\end{Bmatrix} = \frac{1}{\beta}\begin{Bmatrix} \beta\cosh\beta h & \sinh\beta h\\ \beta^2\sinh\beta h & \beta\cosh\beta h\end{Bmatrix}\begin{Bmatrix}\phi_1\\ v_{z1}\end{Bmatrix} \tag{5}$$

The equations can be rearranged further as

$$\begin{Bmatrix}v_{z1}\\ v_{z2}\end{Bmatrix} = \frac{\beta}{\sinh\beta h}\begin{Bmatrix} -\cosh\beta h & 1\\ -1 & \cosh\beta h\end{Bmatrix}\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix} \tag{6}$$

In the context of the layer stiffness matrix method, we need to consider the volume change ($\bar{v}$) as the flux condition at an interface, representing the injected volume via, for example, air-gun source at the interface. In this way, we can easily assemble the layer stiffness matrix in the sense of the finite element method. For this, we need to consider the following relationships:

$$\bar{v}_1 = -v_{z1} \tag{7}$$

$$\bar{v}_2 = v_{z2} \tag{8}$$

The reason for the negative sign in the relationship for the bottom interface (Eq. (7)) is that the volume change is defined positive as it increases, which corresponds to the negative vertical velocity at the bottom interface. Eq. (6) can then be re-written as

$$\frac{\beta}{\sinh\beta h}\begin{Bmatrix} \cosh\beta h & -1\\ -1 & \cosh\beta h\end{Bmatrix}\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix} = \begin{Bmatrix}\bar{v}_1\\ \bar{v}_2\end{Bmatrix} \tag{9}$$

which gives the symmetric acoustic layer stiffness matrix in terms of the velocity potential. Furthermore, the velocity potential can be converted into pressure according to the following constitutive law:

$$p = -i\omega\rho\phi \tag{10}$$

By applying this, we can transform the stiffness matrix in Eq. (9) into the following form in terms of pressure.

$$\frac{i\beta}{\rho\omega\sinh\beta h}\begin{Bmatrix} \cosh\beta h & -1\\ -1 & \cosh\beta h\end{Bmatrix}\begin{Bmatrix}p_1\\ p_2\end{Bmatrix} = \begin{Bmatrix}\bar{v}_1\\ \bar{v}_2\end{Bmatrix} \tag{11}$$

In addition, we can also express the volume change ($\bar{v}$) in terms of the vertical displacement (u_z), and the pressure (p) in terms of the normal traction (σ_z) at the interface. For this, we use a set of constitutive laws of $\bar{v}_1 = -i\omega u_{z1}$, $\bar{v}_2 = i\omega u_{z2}$, $p_1 = -\sigma_{z1}$ and $p_2 = \sigma_{z2}$. Note that we need to impose the negative sign for the bottom interface. Then, we obtain the following stiffness matrix in terms of the vertical displacements.

$$\frac{\rho\omega^2}{\beta\sinh\beta h}\begin{Bmatrix} \cosh\beta h & -1\\ -1 & \cosh\beta h\end{Bmatrix}\begin{Bmatrix}u_{z1}\\ u_{z2}\end{Bmatrix} = \begin{Bmatrix}\sigma_{z1}\\ \sigma_{z2}\end{Bmatrix} \tag{12}$$

As shown in Eqs. (9), (11) and (12), three versions for the acoustic layer stiffness matrix are available and are related to each other via the appropriate constitutive laws. It should be noted that the version with the vertical displacement (Eq. (12)) is in a form to be readily assembled with the soil layer stiffness matrix in the finite element sense without any additional condition to satisfy. On the other hand, the other two versions in Eqs. (9) and (11), with velocity potential and pressure, respectively, require additional interface conditions to satisfy. Such conditions are the flux continuity at interfaces, as given below in Eqs.