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Analysis of Dampers for Stay Cables Using Non Linear Beam Elements

Jean-Marc Battini

Department of Civil and Architectural Engineering, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden

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ABSTRACT

This paper presents a numerical approach to evaluate the damping properties of a stay cable with an external viscous damper. The idea is to model the cable by using non-linear corotational beam elements and to study small vibrations around the static deformed equilibrium configuration. This gives a complex eigenvalue problem from which the modal damping ratios can be calculated. The performance of the proposed method is assessed through two numerical applications. Compared with the analytical methods based on differential equations widely used in the literature, the proposed non-linear finite element approach has the advantages that the effect of the sag is considered in an accurate way and that there is no limitation regarding the number and the value of the structural parameters that can be introduced in the model.

1. Introduction

The problem of defining an optimal external damper for cables in cable-stayed bridges has been the subject of intense research the last decades. Two key contributions in this topic are the works of Uno et al. [1] who introduced the notion of non-dimensional damping coefficient and the work of Pacheco et al. [2] who presented the universal curves relating the modal damping ratio with damper size, damper location, mode number and cable parameters. These works were followed by Krenk [3] who proposed accurate asymptotic approximation of the damping ratio. In all these contributions, the cable is considered as a horizontal taut without bending stiffness. The effects of the inclination of the cable, the sag and the flexural rigidity were introduced and studied by Tabatabai and Mehrabi [4], Krenk and Nielsen [5] and Hoang and Fujino [6]. The influence of the stiffness at the supports and at the damper support was considered by Krenk and Hogsberg [7] and Fujino and Hoang [8]. Other aspects, such as three dimensional vibrations (Yu and Xu [9], Xu and Yu [10]) and clamped supports (Main and Jones [11,12]) were also studied. Alternative designs with two dampers (Hoang and Fujino [13]) and horizontal damper at a support (Jiang, Li and Lu [14]) were proposed. In most of these studies, the mathematical solution was based on the differential equation of the problem and a complex eigenvalue analysis. A different approach was proposed by Cheng et al. [15] and Fournier and Cheng [16]. The idea was to model the cable using finite beam elements and to determine the damping by applying a displacement perturbation and studying the free vibrations. Finally, the works of Wang et al. [17] and Weber et al. [18] who proposed procedures to determine an optimal damper by considering several vibrations modes as well as the book of de Sá Caetano [19] should be mentioned.

The purpose of this paper is to introduce a different approach to determine the modal damping coefficients for stay cables equipped with external dampers. The idea is to use a non-linear finite element model with corotational 2D beam elements. First, the static deformed equilibrium configuration of the cable subjected to its own weight is calculated. Then, small vibrations are considered and a linearization of the dynamic equilibrium equations around the deformed static equilibrium position is performed. This leads to a complex eigenvalue analysis from which the damping ratio from each mode can be obtained. The idea of using discrete models to investigate the placement and size of discrete dampers was also used by Main and Krenk in the context of multi-story shear buildings [20]. In that case, both the stiffness and mass matrices of the structure are constant and a different numerical method was used. An approximate solution was obtained as an interpolation between the solutions of two limiting eigenproblems: the undamped eigenproblem and the constrained eigenproblem in which each damper is replaced with a rigid link.

Compared with the previous finite element approaches proposed in [15,16], the present methodology does not require to first excite the model with a certain mode shape and then to calculate and process the response by time integration. Compared with the solutions based on differential equations, one advantage of the present methodology is that the static deformation of the cable (i.e. the sag) can be considered in a very accurate way. Another advantage of the finite element approach is that there is no restriction regarding the model of the cable. For example, there is no limitation regarding the number and the position of the external dampers or regarding the value of the flexural stiffness; the boundary conditions can be easily changed from simply supported to

E-mail address: jeanmarc@kth.se.

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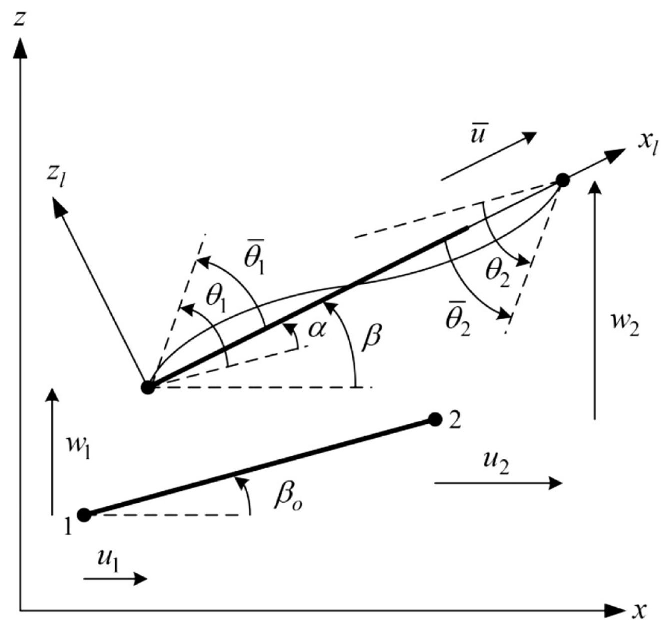

Fig. 1. Cotorational beam element.

clamped; the stiffness at the cable support and at the damper support can be introduced without any difficulties.

The organization of the paper is as follows. In Section 2, the corotational 2D beam non-linear finite element is briefly described. This formulation is not new and more details can be found in the literature. The numerical method is addressed in Section 3. First, the procedure to calculate the static deformed equilibrium configuration of the cable is presented. Then, the linearization of the dynamic equilibrium equations and the calculation of the modal damping ratios using an eigenvalue analysis are explained. Numerical applications are presented in Section 4 and conclusions are derived in Section 5.

2. Finite element formulation

The purpose of this section is to describe briefly the corotational non-linear 2D beam formulation used in the paper. The details regarding the derivation of the element can be found in [21].

The idea of the corotational method, see Fig. 1 is to decompose the motion of the element in two steps. The first step is a rigid body motion defined by the global translation (u_1, w_1) of the node 1 and the rigid rotation α. This rigid motion defines a local coordinate system (x_l, z_l) which continuously rotates and translates with the element. The second step consists of a deformation in the local coordinate system. Assuming that the length of the element is properly chosen, the deformational part of the motion is always small relative to the local axes. Consequently, the local deformations can be expressed in a simple way.

The vectors of global and local nodal displacements are defined by

$$\mathbf{p}_g = [u_1 w_{11} u_2 w_{22}]^{\mathrm{T}} \quad \mathbf{p}_l = [\bar{u} \bar{\theta}_1 \bar{\theta}_2]^{\mathrm{T}}$$

The local displacements are calculated by using

$$\bar{u} = l_n - l_o$$
$$\bar{\theta}_1 = \theta_1 - \alpha$$
$$\bar{\theta}_2 = \theta_2 - \alpha$$

where l_o and l_n denote the initial and current lengths of the element and $\alpha = \beta - \beta_o$.

Differentiation of the above equations gives the transformation matrix **B** as

$$\delta \mathbf{p}_l = \mathbf{B} \mathbf{p}_g \quad \mathbf{B} = \begin{bmatrix} -c & -s & 0 & c & s & 0 \\ -s/l_n & c/l_n & 1 & s/l_n & -c/l_n & 0 \\ -s/l_n & c/l_n & 0 & s/l_n & -c/l_n & 1 \end{bmatrix}$$

with c = $\cos\beta$ and s = $\sin\beta$.

The relation between the local internal force vector $\mathbf{f}_l$ and the global one $\mathbf{f}_g$ is obtained by equating the virtual work in both systems:

$$\mathrm{V} = \delta \mathbf{p}_g^{\mathrm{T}} \mathbf{f}_g = \delta \mathbf{p}_l^{\mathrm{T}} \mathbf{f}_l$$

which by introduction of the transformation matrix **B** gives

$$\mathbf{f}_g = \mathbf{B}^{\mathrm{T}} \mathbf{f}_l$$

Differentiation of the above equation gives after some work the global tangent stiffness matrix as

$$\mathbf{K}_g = \mathbf{B}^{\mathrm{T}} \mathbf{K}_l \mathbf{B} + \frac{\mathbf{z}\,\mathbf{z}^{\mathrm{T}}}{l_n} N + \frac{1}{l_n^2} (\mathbf{r}\,\mathbf{z}^{\mathrm{T}} + \mathbf{z}\,\mathbf{r}^{\mathrm{T}})(M_1 + M_2)$$

where N, M_1 and M_2 are the components of $\mathbf{f}_l$ and

$$\mathbf{r} = [-c - s\ 0\ c\ s\ 0]^{\mathrm{T}}$$

$$\mathbf{z} = [s - c\ 0 - s\ c\ 0]^{\mathrm{T}}$$

Several alternatives are available for the definition of the local formulation. In the present work, a local shallow arch Bernoulli element is taken. The longitudinal strain is defined as

$$\varepsilon_x = \frac{1}{l_o} \int_{l_o} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] \mathrm{d}x - \frac{\partial^2 w}{\partial x^2} z$$

A linear interpolation is used for the local axial displacement u and a cubic interpolation is used for the local transversal displacement w. The local internal force vector $\mathbf{f}_l$ and local tangent stiffness matrix $\mathbf{K}_l$ are then obtained by taking the gradient and the hessian of the strain energy. This operation is performed analytically in Maple (see e.g. [22]). For the dynamic term, the classical linear Bernoulli mass matrix is taken for the global mass matrix of the element. As shown in [22], with relatively fine meshes, a constant global mass matrix gives very accurate results in non-linear problems.

3. Numerical procedure

3.1. Static deformed configuration

The first step of the numerical procedure is to use the non-linear finite element model to calculate the static deformed configuration of the cable when it is loaded by its own weight. This configuration must give a specific and known tension force T at the upper support. For that, the initial (i.e. without loading) configuration of the cable is a straight line, see Fig. 2. At the left support, the translations are fixed whereas at the right support, only the transversal translation is fixed. The static deformed configuration is then calculated in two equilibrium steps. In the first one, only the tension force T is applied. In the second one, the own weight, modelled as vertical forces at the nodes, is added. It can be observed that due to the low bending stiffness of the cable, it is not possible to apply the own weight to the initial unstressed cable. For the second step, the equilibrium equations are solved using Newton-Raphson iterations. Once the static deformed configuration has been obtained, the axial translation at the right support is fixed in order to perform the dynamic analysis.

3.2. Dynamic analysis

The dynamic model and its parameters are shown in Fig. 2.

Let $\mathbf{u}_o$ denote the nodal displacement vector calculated in the static step. The non-linear equilibrium equations defining the deformed static configuration can be written as