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Critical Inter-Load Spacing for Hogging Moments in Three-Span Bridges

C.C. Caprani

Monash University, Melbourne, Australia

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ABSTRACT

This article outlines the determination of the inter-load spacing between two equal point loads traversing a continuous three-span beam, such that the bending moment at the first interior support is a maximum. Different side-span ratios are considered. Exact closed-form expressions are derived, as well as a simple close approximation, making the results easy to implement in spreadsheets and other software. It is motivated by the use of load models from older codes of practice. Bridge owners often make access decisions for heavy vehicles based on the as-built capacities of bridges and a useful proxy for the capacity is the code load model used for the bridge's design. Some of these older load models require two equal point loads for determining the design hogging moments, such as the 1970 NAASRA code in Australia. As such, where applicable, this work will help facilitate the efficient filtering of problematic bridges for more refined structural analyses when making heavy vehicle permit access decisions.

1. Introduction

1.1. Motivation

For the management of existing bridges, decisions about access rights for heavy vehicles are a daily problem for bridge owners. For each new vehicle configuration to be assessed, it is common practice to use some simple models to first eliminate many bridges that can be considered safe. Attention can then be concentrated on bridges whose capacities are close to the actions from the proposed access vehicle.

For this first stage, simple one-dimensional beam models are used, with appropriate lateral distribution factors, and so on. The actions imposed by the proposed vehicle can then be determined, but these must be compared to an appropriate capacity envelope for the structure. Unless the bridge condition ratings indicate otherwise, the present capacity envelope can be assumed to be that of the as-built bridge, which—at a minimum—is sufficient to resist the load model actions to which the bridge was designed. Thus the contemporary load model can act as a proxy for the load capacity of the bridge. This will be conservative since certainly the bridge (which is now many years in service) was built to be at least as strong as its contemporary load model required. The actions due to this reference load model are then a key component of the decision-making process, at least for bridges that are not marginal. This approach is reasonably common, for example in the US [1] and Australia [2].

Automation of the analyses for a wide range of bridge spans (e.g. 1 to 50 m); configurations (e.g. 1-span, 2-span, and 3-span); and load models, is extremely desirable. This facilitates the ready filtering of bridges needing more refined analysis from the population being managed, in order to make an access decision. In many cases, where the load models consist of single or point loads, closed-form solutions are readily available and obviously highly-suitable for automation.

In some older Australian and US bridge design codes (e.g. [3] and [4]), it is specified that two point loads are to be used to determine the design bending moment over a support. The Australian 1970 NAASRA MS18 load model is a good example, where cl. 2.8.3 specifies

> The lane loadings shown in Fig. 2.3 shall be modified for the design of continuous spans in that the lane loadings shall consist of the loads shown in Fig. 2.3, and, in addition thereto, another concentrated load of equal weight shall be placed in one other span in the series in such a position as to produce maximum negative moment.

The referenced Fig. 2.3 of the code is shown in Fig. 1. The H- and HS- load model series of the AASHTO Standard Specifications [4], which apply to bridges constructed between 1931–2007, also have this requirement for the lane loadings (not vehicle loadings). Calculation of the critical hogging moment in this case is a common issue in the load rating of bridges—see for example, [5, Chapter 4].

Consequently, the problem is then to determine the distance between these two equal point loads that gives the critical bending moment at B, shown below in Fig. 2. This distance is termed the critical Inter-Load Spacing (ILS), and denoted here as S^*. It is useful to note that the critical shear and sagging moments for the MS18 (and AASHTO)

E-mail address: colin.caprani@monash.edu.

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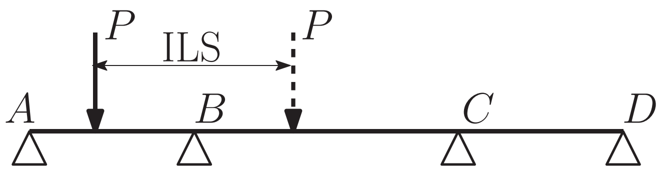

Fig. 1. NAASRA (1970) [3] Lane loading for MS18 load model (similar to AASHTO).

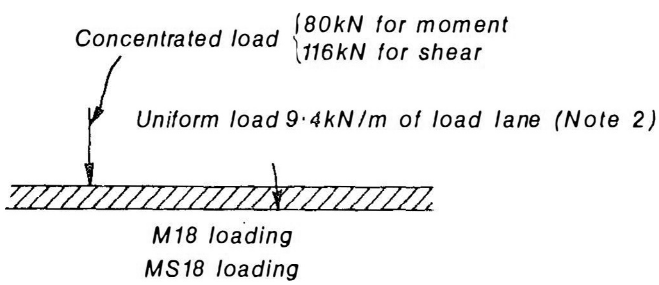

Fig. 2. Three-span continuous beam showing the inter-load spacing (ILS).

lane loading model are defined with single point loads, and so the problem of two point load spacing is particular to hogging moments.

The determination of these critical ILSs for different 3-span bridge configurations and lengths is the subject of this article. A simplified relationship is developed which is suitable for automation.

1.2. Limitation

This work identifies the critical spacing between two equal point loads for determining the worst negative moment in three-span bridges. Two equal point loads are used in some codes of practice around the world. Of course, this work does not relate or cover situations where the load model does not consist of two equal point loads, such as the AASHTO *Manual for Bridge Evaluation* [6]. Typically, it applies in the cases of lane loadings and not vehicle loadings.

1.3. Past work

In the past, influence lines for continuous beams of arbitrary span ratios have been dealt with by the production of books of tables or computer-based analysis. Probably the most comprehensive tabulation of influence lines is the formative work by Anger—first published in German in 1937—which ran to 11 multi-volume editions, as well as in English [7]. More recently, in [8], tables are given for influence ordinates for different span configurations and ratios. Examples are presented to show the workings, but no closed form expressions are developed suitable for computer programming. Alternative computerized approaches have been proposed to compute influence lines directly, e.g. [9] and [10]. However, these approaches are also not closed-form, and require a complete structural analysis in each case. Burgoyne [11] provides an interesting approach to determining influence lines, that does provide a closed-form influence line expression for a given set of parameters, but it is not easily used to generate those of arbitrary span ratios. A good reference for closed-form influence lines is the book by [12]. However, the work of [13] is most relevant here.

While there are some closed-form expressions available for the influence lines, there are none for the critical inter-load spacing for load models such as the MS18 [3] or HS-20 [4] lane loadings. This work address this problem, providing closed-form expressions for the ILS, for a given three-span bridge configuration.

2. Basis

Noting that the bending (hogging) moment at B, M_B, is the load effect of interest, the simplest representation to determine the ILS is to consider the influence line for M_B.

2.1. Three Moment Theorem

To determine the influence line, Clapeyron's *Three Moment Theorem* is extremely useful here. It is given by:

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -6\left(\frac{A_1\bar{x}_1}{l_1} + \frac{A_2\bar{x}_2}{l_2}\right) \tag{1}$$

where A, B, and C are the intermediate supports for any two adjacent spans of lengths l_1 and l_2 of a beam which is continuous over multiple supports. $A_i\bar{x}_i$ is the first moment of the 'free' (equivalent simply-supported beam) bending moment diagram about the left (for $i = 1$) and right (for $i = 2$) hand supports of span i.

2.2. Three span beam influence lines

Next we apply Eq. (1) to determine the influence line. In doing so we make two assumptions:

1. The beam is prismatic: it has constant flexural rigidity, *EI*, across all spans.
2. Beam ends A and D are pinned (see Fig. 2), so that $M_A = 0$ and $M_D = 0$.

Under these assumptions, noting the triangular shape of the free bending moment diagram for a point load of $P_i = 1$, the *Three Moment Theorem*, Eq. (1), for spans *ABC* and *BCD* gives:

$$P_1 l_1^2 \xi_1(1-\xi_1)(1+\xi_1) + P_2 l_2^2 \xi_2(1-\xi_2)(2-\xi_2) + 2M_B(l_1+l_2) + M_C l_2 = 0 \tag{2}$$

$$P_2 l_2^2 \xi_2(1-\xi_2)(1+\xi_2) + P_3 l_3^2 \xi_3(1-\xi_3)(2-\xi_3) + M_B l_2 + 2M_C(l_2+l_3) = 0 \tag{3}$$

where l_i is the length of the ith span and $\xi_i = x_i/l_i$ is the non-dimensional distance of the load along the span. Eliminating M_C from these equations by solving, and setting $P_i = 0$ to determine M_B when the load is positioned on each span ($i = 1,2,3$) in turn, we find the equations of the influence line for each span to be

$$M_B^{(1)} = 2\xi_1(1-\xi_1)(1+\xi_1)\cdot\frac{l_1^2(l_2+l_3)}{K} \tag{4a}$$

$$M_B^{(2)} = \xi_2(1-\xi_2)l_2^2\cdot\frac{3l_2(1-\xi_2) + 2l_3(2-\xi_2)}{K} \tag{4b}$$

$$M_B^{(3)} = -\xi_3(1-\xi_3)(2-\xi_3)\cdot\frac{l_3^2 l_2}{K} \tag{4c}$$

where

$$K = 4(l_1+l_2)(l_2+l_3) - l_2^2.$$

These expressions are similar to those presented in [13]. The expression for M_C can be found similarly.

2.3. Specialization

For the present problem, the ratio between the span lengths is fixed, although the total length, $L = l_1 + l_2 + l_3$, of the bridge may change. Further, the two side spans are equal in length, $l_1 = l_3$. Finally, note from Eqs. (4a) and (4b) that only spans $i = 1,2$ have the same sign, and so M_B will only reach an adverse maximum when load is on these spans, and not on span 3, which is beneficial. Based on this, and also writing the side spans as a multiplier of the centre span,

$$l_1 = l_3 = kl_2 \equiv kl\ , \tag{5}$$