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Shear Strength Models for Reinforced Concrete Slender Beams: A Comparative Study

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ABSTRACT

Shear strength is one of the most widely investigated parameter in reinforced concrete (RC) beams. Numerous papers are published in last 66 years which investigate experimentally and theoretically the shear strength of RC beams without shear reinforcement. In this paper a database on shear strength of 719 RC beams without transverse reinforcement is generated from experimental results published between 1952 and 2018. Well-known shear strength models proposed in the literature are summarized and evaluated through comparison with the experimental results of generated database. It was found that models proposed by Niwa et al. (1986) and Zararis and Papadakis (2001) predicts shear strength of RC beams more accurately than the other models. The former model gives an average strength ratio of 1.05 with a coefficient of variation of 29% while the latter yielded an average strength ratio of 1.10 with a coefficient of variation of 29%. A detailed evaluation in various ranges of parameters revealed that both the models gives unconservative results of shear strength for beams with concrete compressive strength > 90 MPa. Parametric study of the two models within the individual test series showed that model of Niwa et al. does not capture the size effect in RC beams very effectively while the model of Zararis and Papadakis yields uniform results over the whole range of experimental data.

1. Introduction

Shear failure is known to be the most critical failure mode of RC structures, particularly in beams without shear reinforcement. Unlike flexural failure mode, almost no or very little warning occurs, which leads to loss of property and casualties [1,2]. Extensive researches have been conducted to understand the behaviour of RC beams subjected to shear because their prevention from shear failure is of great importance. Mörsch (1909) [3] suggested the first model for shear strength of concrete beams and then it was assumed that shear riddle has been solved. In 1955 and 1956 two roofs of U.S air force base were collapsed under their self-weight having a collapse area of > 900 m^2 [4]. Shear failure in reinforced concrete girders without shear reinforcement was responsible for the collapse of the roofs. Investigations revealed that design of girders in both the cases was strictly as per the design standards of that time. Investigators and engineers of that time realized that their knowledge about the shear strength of RC structures is still limited [5].

Numerous RC beams failing in shear are tested by different researchers which resulted in the development of shear strength prediction models for RC beams without web reinforcement. Besides concrete compressive strength, shear-span-to-depth ratio and percentage of longitudinal reinforcement, size of the member was also recognized as an important factor that affects the shear stress at failure of RC beams without transverse reinforcement [6–10].

The aim of this paper is to conduct a comprehensive evaluation and comparison of the models of shear strength for RC slender beams (shear-span-to-depth ratio > 2.5) through comparison with database of experimental results which has not yet been discussed in literature. A database on shear strength of RC slender beams without shear reinforcement was generated by Reineck et al. (2003) [11]. Later, this database was expanded by joint ACI-DAfStb group as reported by Reineck et al. (2013) [12]. This expanded database included the details of test results of RC beams tested for shear strength which are published up to 2011. Database given in Reineck et al. (2013) [12] reported the beams of normal weight and lightweight concrete tested under point loads or distributed load. In last seven years a significant amount of research papers are published on shear strength of RC beams which are also included in the database given in the present study. The updated database was generated from research papers published between 1952 and 2018. Seven hundred nineteen (719) RC slender beams made with normal weight concrete having rectangular cross-section failing in shear were included in the database. Beams included in the database were simply supported, tested either in three point bending or four

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point bending. Continuous beams or beams with uniformly distributed load were excluded from the database. Similar study has been performed on deep beams [13].

2. Models for shear strength of RC slender beams

In this paper shear strength models proposed in the literature since 1950s are classified into five categories: 1) empirical models; 2) mechanics based approaches; 3) semi-empirical models; 4) numerical models based on finite element approach and; 5) artificial intelligence models. Empirical models referred here are the models which are exclusively derived from the regression analysis of experimental data [14–16]. The models which are obtained purely by the rules of mechanics are referred here as mechanics based approaches [17,18]. Some well-known approaches exists in the literature which are derived on rational or mechanics basis but their coefficients are determined by fitting the equation to shear strength data of experimentally tested beams [19–21]. These models are referred here as semi-empirical models. Eight well-known models which are considered in this study are from the first three categories i.e. empirical models, mechanics based models and semi-empirical models. Though the application of finite element [22–24] and artificial intelligence models [25–27] in structural engineering is becoming increasingly common but require significant skill to apply in a reasonable way. Therefore, these models are not included in this study. Many studies included in the database did not reported the values of longitudinal strains, therefore, modified compression field theories (MCFT) are also excluded from the analysis since they involve the values of longitudinal strains.

2.1. Empirical models

These are derived from the regression analysis of the experimental results without putting much emphasis on a physical explanation. Difference in various empirical models is due to the selection of structure of equation and the coefficients used in the formulation. Two most common examples of the structures selected for shear strength models are

$$v_u = k_1 (f_c')^{k_2} + k_3 \frac{\rho}{a/d} \tag{1}$$

$$v_u = k_1 \left(\frac{f_c' \rho}{a/d} \right)^{k_2} \tag{2}$$

ACI-318-14 [28] adopts shear strength model similar to Eq. (1) while Japanese Building code [29] adopted shear strength model similar to Eq. (2).

Zsutty (1968) [14] proposed an empirical equation to predict the shear strength of reinforced concrete beams by combining the dimensional analysis and regression analysis techniques. It was identified that concrete compressive strength (f_c'), shear-span to depth ratio (a/d) and amount of longitudinal reinforcement are the main factors influencing the shear strength of a member. Size effect was not included in the proposed equation since the experimental database used for regression analysis included the beams up to 500 mm depth.

The following equation was proposed for the prediction of shear strength of RC beams.

$$V_u = 2.2 \left[\left(f_c' \rho \frac{d}{a} \right)^{1/3} \right] bd \tag{3}$$

Niwa et al. (1986) [15] formulated an equation for the prediction of shear strength of RC beams from the statistical analysis of experimental results. Experimental results of Kani (1967) [6] suggested that the ultimate shear stress of RC beams decreased with an increase in effective depth though the other parameters were kept constant. This is called size effect and was incorporated by Niwa et al. (1986) [15] in their equation which was taken to be proportional to $d^{-1/4}$ on the basis of Weibull's weakest link theory.

$$V_u = \left[0.2 (100 f_c' \rho)^{1/3} \left(d^{-\frac{1}{4}} \right) \left(0.75 + 1.4 \frac{d}{a} \right) \right] bd \tag{4}$$

Sarsam and Al-Musawi [16] on the basis of their experimental work and data from literature proposed an equation similar to Zsutty's [14] equation for the prediction of shear strength of RC beams. Authors proposed small changes in Zsutty's [14] model and pointed out that shear strength of RC beams is proportional to $\left(f_c' \rho \frac{d}{a}\right)^{0.38}$ rather than $\left(f_c' \rho \frac{d}{a}\right)^{1/3}$ as proposed by Zsutty and proposed the following equation

$$V_u = 1.8 \left[\left(f_c' \rho \frac{d}{a} \right)^{0.38} \right] bd \tag{5}$$

2.2. Mechanics based models

Zararis and Papadakis (2001) [17] proposed a simple expression which shows that ultimate shear strength of a section is the product of splitting tensile strength of concrete (f_{ct}) and the ratio of neutral axis depth to effective depth of the section $\left(\frac{c}{d}\right)$. Test results of Sabnis and Mirza (1979) [30], Hasegawa et al. (1985) [31] and Bazant et al. (1991) [32] on different sizes of cylindrical disks with constant thickness confirms that up to a certain critical diameter splitting tensile strength decreased as the diameter was increased. Therefore Zararis and Papadakis (2001) [17] considered the size effect in the proposed model which is a function of effective depth and shear-span-to-depth ratio of the member.

$$V_u = \left[\left\{ 1.2 - 0.2 \frac{a}{d} d \right\} \frac{c}{d} f_{ct} \right] bd \tag{6}$$

$$\text{Size factor:} \left\{ 1.2 - 0.2 \frac{a}{d} d \right\} \geq 0.65 \tag{7}$$

$$f_{ct} = 0.3 f_c'^{2/3} \tag{8}$$

$$\left(\frac{c}{d}\right)^2 + 600 \frac{\rho + \rho'}{f_c'} \left(\frac{c}{d}\right) - 600 \frac{\rho + \left(d'/d \right) \rho'}{f_c'} = 0 \tag{9}$$

Tureyen and Frosch (2003) [18] proposed the shear strength model using a free body diagram of a cracked concrete member subjected to axial compression and shear stress as shown in Fig. 1. It was assumed that failure will occur when the principal tensile stress in the concrete above neutral axis (uncracked region) reaches the concrete tensile strength (f_{ct}). Following equation was proposed for the prediction of shear strength.

$$V_u = \frac{2}{3} bc \sqrt{f_t^2 + f_t \frac{\sigma_m}{2}} \tag{9}$$

$$f_t = 0.5 \sqrt{f_c'} \tag{10}$$

$$\sigma_m = 0.625 \sqrt{f_c'} \tag{11}$$

σ_m = stress in extreme compression fibre at cracking moment
c = kd = neutral axis depth of cracked transformed section
$k = \sqrt{2\rho m + (\rho m)^2} - (\rho m)$
$m = E_r / E_c$
E_r = modulus of elasticity of reinforcement
E_c = modulus of elasticity of concrete = $4700\sqrt{f_c'}$ MPa
$\rho = A_r / bd$
d = effective depth of the section