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Bifurcation and large-deflection analyses of thin-walled beam-columns with non-symmetric open-sections

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ABSTRACT

Non-symmetric thin-walled open-sections are used extensively in metal structures. The behaviour of such sections is usually complex because their shear centre and centroid do not coincide, and as a result, they are often susceptible to lateral-torsional and/or flexural-torsional buckling. This paper develops an efficient and robust computational framework for practical application, including a refined beam-column element for non-symmetric sections, a cross-section analysis algorithm for determining pertinent properties of open-sections, and an Updated-Lagrangian (UL) solution method for large deflection analysis. Details of the derivation of element stiffness matrices and the related numerical procedures are provided. Several examples are given that demonstrate accurate results from its implementation within the educational structural analysis software MASTAN2.

1. Introduction

Thin-walled open-sections, such as those shown in Fig. 1, are extensively used in metal structures because of their material efficiency and ease in manufacturing, with the latter often promoting the utilization of non-symmetric sections. Members with these sections are usually weak in resisting torsion and minor-axis bending. As a result, they are susceptible to buckling in a lateral-torsional mode under bending, in a flexural-torsional mode under compression, or in a coupled mode under eccentric axial load. The behaviour of these non-symmetric sections is more complex because its shear centre does not coincide with its centroid. As a preferred design method for handling such sections, simulation-based design approaches come to the forefront – approaches in which more factors known to influence system stability are modelled directly within the analysis, and thereby require that a smaller number of prescriptive equations be employed in the design process. The key to such approaches, however, is a robust, efficient, and reliable computational analysis method that accurately models member and system behaviour. To this end, a refined warping element for the bifurcation and large-deflection analysis of beam-columns with arbitrary thin-walled open-sections is presented in this paper.

Theoretical solutions for calculating the buckling strengths of slender thin-walled members with idealized boundary conditions were studied extensively by the 1950s and1960s [1–4]. Based on these analytical methods, some design codes and guidelines, such as BS 5950–5 [5], provided empirical equations for determining the buckling strength of cold-formed members with open-sections. In more modern codes, such as AISC 360–2016 [6], CoPSC 2011 [7], and Eurocode 3–1 [8], design approaches are adopted whereby the structural response of members can be directly simulated when confirming buckling strengths under design loads.

Research on the stability of thin-walled beam members was initiated when the linear theory of non-uniform torsion for elastic beams was proposed by Vlasov [4] in 1962. This topic has received continuous attention over the past 50-plus years and has been studied by several researchers. Such investigators include Bradford and his associates [9–12], Kitipornchai and Trahair [13–15], Yang and his associates [16,17], Rasmussen and his research team [18,19], and several others [20–23]. These researchers have assumed the section to be doubly-symmetric, and thus the effects caused by the shear centre and the centroid not being coincidental are not included in their element formulations. As reported by Mohri *et al.* [24], the buckling strength of a slender beam with a mono-symmetric I-shape section can be dramatically over-estimated (by as much as a factor of two) when such conventional symmetric warping elements are used.

More recently, researchers, such as Chan and Kitipornchai [25], Shakourzadeh *et al.* [26], Kim [27], Hsiao and Lin [28], Pi and Bradford [29], Saade *et al.* [30] and Machado [31] have formulated beam-column elements with a warping degree of freedom (DOF) for members with general thin-walled sections. Because these elements were

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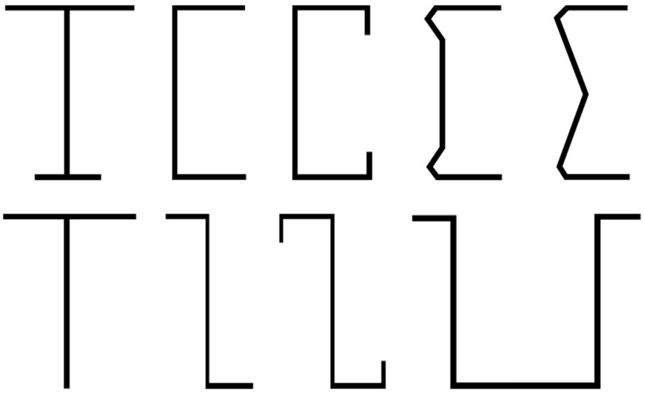


Fig. 1. Examples of non-symmetric thin-walled open-sections.

developed assuming the load is applied only at the shear centre, they are inconsistent with the conventional elements that adopt the centroid as the origin for the element's local axes. In the work presented in this paper, element end nodes are also initially located at the shear centre as a means for simplifying the formulations. After the derivations, however, they are then transferred to the geometric centroid.

An accurate calculation of the cross-section properties, especially for the key parameters related to non-symmetric sections, such as the location of the shear centre (z_s and y_s) and the Wagner coefficients [32] (β_y, β_z and β_ω), is essential. For the simple shapes of thin-walled sections, such as mono-symmetric-I, T-, and L-shapes, the analytical expressions of the Wagner coefficients can be easily derived and are given by Ziemian [33]. These expressions, however, tend to be very complicated and are usually difficult to apply in practical design methods. Although cold-formed sections with irregular, asymmetric and complex shapes are commonly adopted in light load-bearing structural systems, such as light gauge façade framing and non-load bearing roof systems, their Wagner coefficients are in most circumstances nearly impossible to represent with closed-form mathematical expressions. With this in mind, a numerical algorithm is developed herein for arbitrary open-sections and elaborated in detail, thereby potentially promoting wider application of innovative section shapes in more routine design.

Large displacement analyses are well established in the literature, with several based upon the Updated-Lagrangian (UL) formulation [34]. Such an approach allows for moderately large deflections and rotations, and can take into account the coupled fields of deformations due to axial, bending, shear, torsion and bi-moment actions. The equilibrium conditions of the elements are determined based on the last-known configurations during each analysis step. Since the rotations in three-dimensional space are non-vectoral, consecutive rotations are applied as a means for minimizing the additive error in the analysis – a procedure that was originally reported by Argyris *et al.* [35] and has proven to be effective by several researchers [36–38].

In summary, this paper provides a detailed derivation of the element stiffness formulations for non-symmetric cross-sections. A numerical algorithm for providing section properties for arbitrary thin-walled open-sections is developed. The kinematic motion is based on the UL formulation and is discussed. Finally, several examples are given that demonstrate the accuracy of the results from its implementation within a new version of MASTAN2 [39].

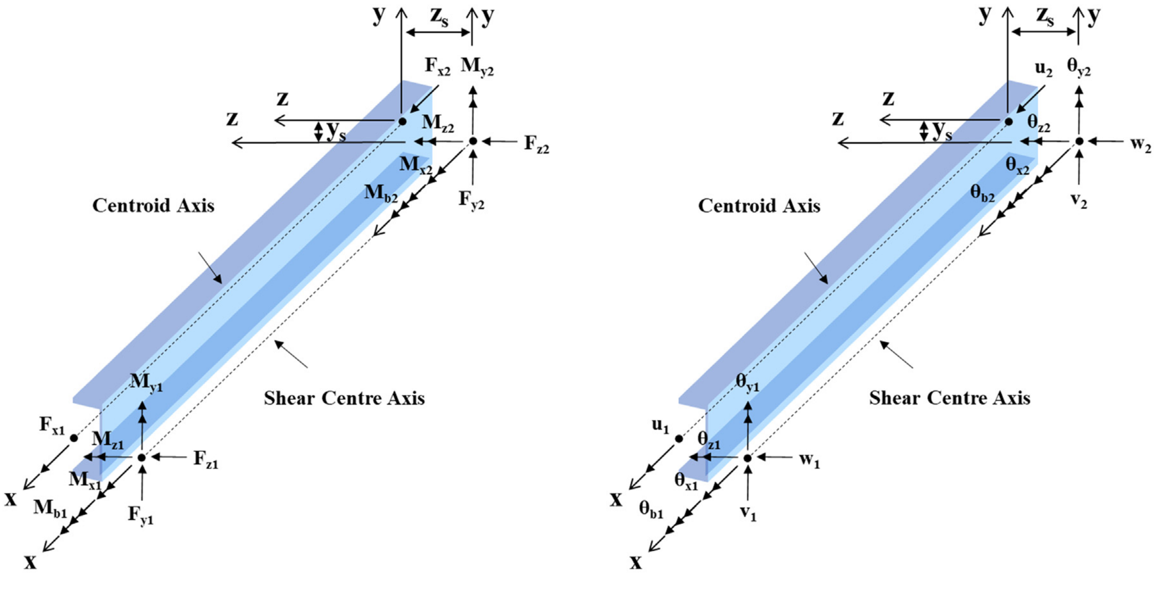
2. Assumptions

To simplify the element formulation, the following assumptions are made: (1) Distortion of the cross-section is not considered; (2) Applied loads are conservative; (3) The material is elastic, isotropic, and homogeneous; (4) Local buckling is not modelled; and (5) Strains are small, but the displacements and deflections can be large. In general, these assumptions should not limit the application of the proposed element in a large majority of engineering design practice.

3. Beam-column element formulations

3.1. Element reference axes

An additional warping degree of freedom (DOF) is included in the proposed three-dimensional line-element formulation. As a result, there are seven DOFs at each node of an element end, and therefore, the total number of DOFs for the element is fourteen (see Fig. 2(a)). There are two reference local axes per element, one located at the centroid and the other at the shear centre. In order to simplify the formulations, the rotations, translations, and warping deformations due to moments, shears, and bi-moments are defined relative to the shear centre axis,



(a) Local DOFs

(b) Local element forces

Fig. 2. Deformations and forces in the element local axes.