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Constrained spline Finite Strip Method for thin-walled members with open and closed cross-sections

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ABSTRACT

The present paper aims at introducing the constrained spline Finite Strip Method (csFSM). The proposed approach is basically a spline Finite Strip Method (spline FSM) that allows the modal decomposition. Similarly to the constrained Finite Strip Method (cFSM), some mechanical assumptions are made in order to constrain the general spline FSM model to buckle in specific modes, for example to enforce the member to buckle in the local-plate mode, or distortional mode. Derivation of matrices that define the distortional (D) and global (G) modes for thin-walled members with unbranched open and closed cross-sections is the main objective of this paper. To define these subspaces, a standard practice is followed which consists in forming $\mathbf{R}_{GD}$, the constraint matrix of the combined *GD* space, then, $\mathbf{R}_G$ and $\mathbf{R}_D$ the constraint matrices of pure global and distortional buckling modes, respectively. Mechanical criteria are used to derive $\mathbf{R}_{GD}$ and $\mathbf{R}_G$ matrices, while orthogonality conditions are used to derive $\mathbf{R}_D$ matrix. The implementation of the mechanical criteria is done by using FEM procedure rather than the cFSM one. Moreover, some practical aspects on how to constrain a spline FSM model are also discussed, including how to force the torsional mode of closed cross-sections. Numerical examples of modal decomposition are provided for a column - beam problem, with standard boundary conditions. The distortional and global buckling loads obtained are found to be in good agreement with those calculated via the cFSM and the Generalized Beam Theory (GBT). The paper concludes with a discussion on the applicability of csFSM in cold-formed steel member design.

1. Introduction

In order to examine and understand the complicated behaviour of a structural member, it is generally preferable to use a practical method, which consists in decomposing the complex phenomenon in less complicated ones. This complex phenomenon is transformed into a series of phenomena easier to understand. As a consequence, the deformations of a beam or a column member with thin-walls are frequently classified into more straightforward but still much more significant deformation categories, i.e. global, distortional, local-plate and other categories, which present some distinctive characteristics of the deformations.

The deformations of thin-walled members, such as cold-formed steel ones, are often categorized into specific classes; they can be global (G), distortional (D), local-plate (L), shear (S) and transverse extension (T). Classes *G*, *D* and *L* are the most significant ones in several practical cases.

It was found that the modal decomposition of the behaviour of a thin-walled member is particularly advantageous in understanding and investigating the stability behaviour. This behaviour, which is due to its thin-walled nature, i.e. high slenderness of the structure, is encountered in many practical situations. The classification is also employed in capacity prediction; it can be seen either implicitly or explicitly in current design standards used for cold-formed steel structures, as can be seen in [1,2]. Presently, buckling mode decomposition for thin-walled members may easily be performed by means of the Generalized Beam Theory (GBT) and the constrained Finite Strip Method (cFSM). The Generalized Beam Theory has been used to show that buckling deformations can be formally dealt with in a modal nature that automatically distinguishes between the global, distortional, local, and other deformations, as can be seen in [3–6]. Moreover, the Generalized Beam Theory (GBT) involves the separation of the fundamental deformation modes; it makes it possible to calculate the pure buckling mode and to measure the modal participation in coupled modes.

Similarly to the GBT, the constrained Finite Strip Method (cFSM) is also capable of decomposing and identifying a mode. The constrained Finite Strip Method (cFSM) was initially suggested and formulated for the semi-analytical Finite Strip Method (FSM) with sine-cosine longitudinal shape functions which are equivalent to locally and globally

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pinned-pinned end restraints of thin-walled beams or columns [7–9]. Afterwards, alternative end conditions were also considered in [10,11], by using the trigonometric interpolation functions introduced by Bradford and Azhari [12]. Later on, the method was made more general in order to render it capable of handling general cross-sections [13,14]. Both the Generalized Beam Theory (GBT) and the constrained Finite Strip Method (cFSM) are readily accessible, and can be carried out within the free-to-use packages GBTUL [15] and CUFSM [16].

Casafont et al. [17,18] applied the constraining technique to the shell finite element method, within the context of a commercial finite element code, in order to investigate the decomposition of buckling modes that is based on a shell model. Djafour et al. made an attempt to first simplify the derivation of the constraint matrix, which defines the combined buckling mode space formed by the global and distortional modes [19], and then to extend the constrained Finite Strip Method (cFSM) to study prismatic members with arbitrary cross-sections [20]. Furthermore, Becque [21] proposed a new modal decomposition method, which is based on the polarization of the modal output towards plate bending and membrane energies. The constrained finite element analysis of thin-walled structural members was introduced in [22,23] and was then applied to the linear buckling analysis of thin-walled members with arbitrary restraints, loadings and holes [24,25].

The aim of the present paper is to improve the spline Finite Strip Method (spline FSM) by introducing the buckling mode decomposition capacity. The method suggested here is fully implemented in the context of the spline FSM, and therefore it can be called a constrained spline Finite Strip Method (csFSM). For the sake of a compact and clear presentation, this paper focuses only on the global and distortional modes for thin-walled members with unbranched open and closed cross-sections, including the torsional mode of closed cross-sections. The formulation of the constraining technique is detailed for members with standard boundary conditions. Some basic examples are considered for validation and the computed global (G) and distortional (D) buckling curves are compared with those given by the conventional spline FSM, constrained FSM and GBT. It is to note that some results (namely: results on the calculation of *GD* elastic buckling loads for simply supported and clamped-clamped members) have already been presented in the 6th International Conference on Coupled Instabilities in Metal Structures [26]. This paper, thus, can be regarded as a modified and significantly extended version of [26].

It is worth mentioning that a procedure for the calculation of pure buckling modes by means of spline FSM linear buckling analysis has been recently presented in [27,28]. The main difference between this procedure and the procedure proposed in the present paper is the fact that in [27,28] the constraint matrix of the combined *GD* space has been derived by using GBT basic assumptions and separated into *G* and *D* space by using warping functions derived from GBT cross-sectional analysis. Moreover, to deal with general boundary conditions trigonometric functions of Bradford and Azhari [12] have been used.

To start with and help the reader better understand the proposed procedure, few words about the spline FSM are in order.

2. The spline Finite Strip Method

The spline Finite Strip Method (spline FSM) was developed from the semi-analytical Finite Strip Method (FSM) originally derived by Cheung [29]. The Finite Strip Method (FSM) was based on harmonic functions, and proved to be an efficient tool for analysing members with constant geometrical properties along the longitudinal direction. The spline Finite Strip Method (spline FSM) complemented the semi-analytical Finite Strip Method (FSM) by allowing more complex types of loading and support conditions since it uses a more comprehensive set of displacement functions based on splines [30].

In the formulation of the spline FSM, a typical thin-walled member is divided into ns strips having all the length, L, of the member (Fig. 1). The intersection line of two connecting strips is called nodal line or simply node, and the total number of nodes is denoted as nn. As shown in Fig. 1, each nodal line is divided into nm equal intervals by $nm + 1$ section knots. To define the spline function completely over the length of the strip, two additional section knots are required which gives a total of $nm + 3$ section knots per node, numbered from -1 to $nm + 1$. Each section knot has four degrees of freedom (DOFs) corresponding to the two membrane (in-plane) DOFs, u and v, and the two flexural (out-of-plane) DOFs, w and θ. Consequently, the total number of degrees of freedom for a folded plate system is $(4 \times nn \times (nm + 3))$, in a spline finite strip analysis.

To derive the matrix formulation, two left-handed coordinate systems are used: global and local. The global coordinate system is denoted as: X-Y-Z, with the Y axis parallel with the longitudinal axis of the member. The local system, which is always associated with a strip, is denoted as x-y-z. The x axis is parallel with the plate element, and perpendicular to the member longitudinal axis, the y axis is parallel with Y, and the z axis is perpendicular to the x-y plane.

The assumed displacement function of the strip is a product of a B3-spline function along the longitudinal direction and a Hermitian function in the transverse direction. The transverse Hermitian function is a linear polynomial for membrane displacements (u and v) and a cubic polynomial for flexural displacements (w and θ). The displacement functions are given by

$$u = [N_1 \quad N_2]\begin{bmatrix} \boldsymbol{\psi}_{ui} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\psi}_{uj} \end{bmatrix}\begin{Bmatrix} \boldsymbol{u}_i \\ \boldsymbol{u}_j \end{Bmatrix} \tag{1}$$

$$v = [N_1 \quad N_2]\begin{bmatrix} \boldsymbol{\psi}_{vi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\psi}_{vj} \end{bmatrix}\begin{Bmatrix} \boldsymbol{v}_i \\ \boldsymbol{v}_j \end{Bmatrix} \tag{2}$$

$$w = [N_3 \quad N_4 \quad N_5 \quad N_6]\begin{bmatrix} \boldsymbol{\psi}_{wi} & & & \mathbf{0} \\ & \boldsymbol{\psi}_{\theta i} & & \\ & & \boldsymbol{\psi}_{wj} & \\ \mathbf{0} & & & \boldsymbol{\psi}_{\theta j} \end{bmatrix}\begin{Bmatrix} \boldsymbol{w}_i \\ \boldsymbol{\theta}_i \\ \boldsymbol{w}_j \\ \boldsymbol{\theta}_j \end{Bmatrix} \tag{3}$$

The parameters N_1, N_2, N_3, N_4, N_5 and N_6 are transverse shape functions and $\boldsymbol{\psi}_{ui}$, $\boldsymbol{\psi}_{vi}$, $\boldsymbol{\psi}_{wi}$, $\boldsymbol{\psi}_{\theta i}$, $\boldsymbol{\psi}_{uj}$, $\boldsymbol{\psi}_{vj}$, $\boldsymbol{\psi}_{wi}$ and $\boldsymbol{\psi}_{\theta j}$ are B3-spline representations. The expressions for the transverse shape functions are given by

$N_1 = 1-\bar{x}$
$N_2 = \bar{x}$
$N_3 = 1 - 3\bar{x}^2+2\bar{x}^3$
$N_4 = x(1 - 2\bar{x} + \bar{x}^2)$
$N_5 = 3\bar{x}^2-2\bar{x}^3$
$N_6 = x(\bar{x}^2 - \bar{x})$ in which $\bar{x} = x/b$ and b is the width of the strip.

The B3-spline representations have $nm + 3$ local terms and they are defined by uniform B3 spline functions, $\psi_k(y)$ and amended B3-spline functions, $\bar{\psi}_k(y)$. Such modified functions are required to represent the specified boundary conditions for each DOF and concerns the three first and the last ones [31]. For instance, the B3-spline function for u DOF of node i can be written as follows

$$\boldsymbol{\psi}_{ui} = \left[\bar{\psi}_{-1}^{ui} \;\; \bar{\psi}_{0}^{ui} \;\; \bar{\psi}_{1}^{ui} \;\; \psi_{2}^{ui} \;\; \cdots \;\; \psi_k \;\; \cdots \psi_{nm-2}^{ui} \;\; \bar{\psi}_{nm-1}^{ui} \;\; \bar{\psi}_{nm}^{ui} \;\; \bar{\psi}_{nm+1}^{ui} \right]$$

$\boldsymbol{\psi}_{vi}$, $\boldsymbol{\psi}_{wi}$, $\boldsymbol{\psi}_{\theta i}$, $\boldsymbol{\psi}_{uj}$, $\boldsymbol{\psi}_{vj}$, $\boldsymbol{\psi}_{wi}$ and $\boldsymbol{\psi}_{\theta j}$ are described in the same way. The equal section B3-spline function and its first derivatives at the section knots are well-known, and are given in [31,32]. The plate bending and membrane behaviours are completely uncoupled. Thus, the displacement field of a strip, i.e., $\mathbf{d}^{(s)}$, in the local $x - y - z$ coordinate system, which is interpolated from the local degrees of freedom (DOFs) of its two nodes, can be written as follows

$$\mathbf{d}^{(s)} = [\boldsymbol{u}_i \; \boldsymbol{v}_i \; \boldsymbol{u}_j \; \boldsymbol{v}_j | \boldsymbol{w}_i \; \boldsymbol{\theta}_i \; \boldsymbol{w}_j \; \boldsymbol{\theta}_j]^{\mathrm{T}}$$

or in partitioned form

$$\mathbf{d}^{(s)} = [\mathbf{d}_{\mathrm{M}}^{\mathrm{T}} \, \mathbf{d}_{\mathrm{F}}^{\mathrm{T}}]^{\mathrm{T}}$$

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